Meter's Influence on Theoretical and Corpus-Derived Harmonic Grammars

Christopher Wm. White
The University of Massachusetts Amherst

Introduction

Harmonic changes both inform how we hear metrically strong beats, and contribute to the ways composers express metric emphasis, at least so says a certain consensus of music researchers and pedagogues. Even in previous centuries, theorists such as Koch and Kirnberger suggested that harmonic changes should align with metric emphases,[1] and in recent decades several researchers have constructed theories in which tonal changes at various time scales should ideally support the music's meter.[2] These kinds of insights have been supported by music cognition research which finds that harmonic changes or fluctuations in tonal stability influence participants' understanding of a passage's meter.[3] Even music-theory pedagogy encourages students to consider using harmonic changes to support a meter: when teaching chorale-style model composition, textbooks instruct

[1] Danuta Mirka, *Metric Manipulations in Haydn and Mozart: Chamber Music for Strings, 1787–1791* (New York: Oxford University Press, 2009).

[2] See Maury Yeston, *The Stratification of Musical Rhythm* (New Haven: Yale University Press, 1976); and Fred Lerdahl and Ray Jackendoff, *A Generative Theory of Tonal Music* (Cambridge, MA: MIT Press, 1983).

[3] See Jon B. Prince, William F. Thompson, and Mark A. Schmuckler, "Pitch and Time, Tonality and Meter: How Do Musical Dimensions Combine?" *Journal of Experimental Psychology* 35, no. 5 (2009): 1598–1611; and Justin London, Tommi Himberg, and Ian Cross, "The Effect of Structural and Performance Factors in the Perception of Anacruses," *Music Perception* 27, no. 2 (2009): 103–20.

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students to write "changes of chord [that] support the meter,"[4] to "avoid repeating a chord from a weak to a strong beat,"[5] and to use "the speed of harmonic change" to reinforce the meter.[6]

In sum, while there may be many factors that contribute to how music expresses accent, some portion of that task is done by harmony changes.[7] But, how much of this obligation determines the kinds of chords that follow other chords? How much does meter dictate harmonic syntax? While studies of harmonic grammars (especially in the corpus and computational domains) have investigated the actual observed chord-to-chord changes of analytical annotations,[8] the surface

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[4] Edward Aldwell and Carl Schachter, *Harmony and Voice Leading,* 4th ed. (Boston: Schirmer, 2011), 93.

[5] Aldwell and Schacter, *Harmony and Voice Leading*, 93.

[6] Steven Laitz, *The Complete Musician: An Integrated Approach to Tonal Harmony, Analysis, and Listening,* 3th ed. (New York: Oxford University Press, 2012), 27.

[7] For the purposes of this study, "accent" will mean a musical moment that is heard as relatively stronger or more marked than other surrounding moments. See Grosvenor Cooper and Leonard B. Meyer, *The Rhythmic Structure of Music* (Chicago: The University of Chicago Press, 1960). "Meter," on the other hand, is a phenomenon that a) arises from a series of consistently paced accents [see Lerdahl and Jackendoff, *A Generative Theory of Tonal Music*; and Harald Krebs, *Fantasy Pieces: Metrical Dissonance in the Music of Robert Schumann* (New York: Oxford University Press, 1999)], b) involves a listener expecting that pacing to continue into the future [see Justin London, *Hearing in Time: Psychological Aspects of Musical Meter* (New York: Oxford University Press, 2004)], and c) groups adjacent pulses by either twos or threes to form a hierarchy of stronger and weaker pulses [see Richard Cohn, "Complex Hemiolas, Ski-Hill Graphs and Metric Spaces," *Music Analysis* 20, no. 3 (2001): 295–326]. For instance, hearing accents on an alternating quarter-note pulse could lead someone to hear that pattern in a duple meter. I will refer to a series of regular accents that might support a meter as "metrical accents."

[8] See Martin Rohrmeier and Ian Cross, "Statistical Properties of Tonal Harmony in Bach's Chorales," in *Proceedings of the 10th International Conference on Music Perception and Cognition* (Sapporo, 2008), 619–27; David Temperley, "A Statistical Analysis of Tonal Harmony," last modified 2009, davidtemperley.com/kp-stats/; Trevor DeClercq and David Temperley, "A Corpus Analysis of Rock Harmony," *Popular Music* 30, no. 1 (2011): 47–70; Dmitri Tymoczko, *A Geometry Of Music: Harmony and Counterpoint in the Extended Common Practice* (New York: Oxford University Press, 2011); and John Ashley Burgoyne, *Stochastic Processes and Database-Driven Musicology* (PhD diss., McGill University, 2012).

events of some symbolic musical representation,[9] or even of the sound signal itself,[10] this study proposes a somewhat different tactic.

In what follows, I outline some properties of common-practice chord changes, showing that certain chords tend to accompany metrical accents while others appear on weak beats. More specifically, I will show that relatively large changes in pitch classes (here, "pcs") tend to accompany stronger accents, and that I, IV, and V triads tend to occur on relatively strong pulses. I build a model that generates chord progressions using these corpus-derived tendencies/preference rules, and I validate the resulting progressions against other models and corpora by correlating the resulting transition matrixes against one another. I conclude by suggesting that these results might yield insights into the relationship between harmony and meter, that these relationships might generalize across musical styles, and that these findings might interact with traditional theoretical models of chord "function."

The Kinds of Harmonic Changes that Accompany Metric Pulses: A Corpus Analysis

Several corpus analyses have suggested that different types of chords and scale degrees occur on strong versus weak beats, with research showing that structures which appear more frequently in a corpus tend

[9] See Ian Quinn, "What's 'Key for Key': A Theoretically Naive Key–Finding Model for Bach Chorales," *Zeitschrift der Gesellschaft für Musiktheorie* 7, no. ii (2010): 151–63; Ian Quinn and Panayotis Mavromatis, "Voice Leading and Harmonic Function in Two Chorale Corpora," in *Mathematics and Computation in Music*, ed. Carlos Agon et al. (New York: Springer, 2011), 230–40; Christopher White, "Changing Styles, Changing Corpora, Changing Tonal Models," *Music Perception* 31, no. 2 (2014): 244–53; Christopher White, "A Corpus-Sensitive Algorithm for Automated Tonal Analysis," in *Mathematics and Computation in Music*, ed. T. Collins, D. Meredith, and A. Volk (New York: Springer, 2015), 115–21; and Florian Colombo et al., "Algorithmic Composition of Melodies with Deep Recurrent Neural Networks," in *Proceedings of the First Conference on Computer Simulation of Musical Creativity*, last modified June 17–19, 2016, https://csmc2016.wordpress.com/ proceedings/.

[10] See Matthais Mauch et al., "The Evolution of Popular Music: USA 1960–2010," *Royal Society Open Science*, published May 6, 2015, http://rsos.royalsocietypublishing.org/content/2/5/150081; and Bruno Mesz et al., "The Evolution of Tango Harmony, 1910–1960," in *Mathematics and Computation in Music*, ed. Emilio Lluis-Puebla, Octavio A. Agustín-Aquino, and Mariana Montiel (New York: Springer, 2017), 291–97.

to align with that corpus's metrically emphasized beats.[11] Similarly, the current study's corpus analysis investigates the kinds of chord changes that tend to occur between different beat strengths, and whether different chord types tend to occur on strong versus weak beats. In particular, this analysis will show 1) greater metric stress indicates greater amounts of pc change, and 2) different types of chords occur on strong beats versus weak beats.

The Corpora

These analyses rely on two corpora, both of which were derived from the xml-encoded pieces in the music21 database.[12] What I will refer to as the *tonal corpus* comprised quadruple-meter, major-mode pieces from Western-European composers who lived between 1650 and 1900 represented in this dataset: Bach, Beethoven, Haydn, Mozart, Schubert, Schumann, and Verdi. When investigating the scale-degrees of these pieces, I use only the first eight measures of these files to avoid modulations, and use the file's tagged key to transpose those measures to C (here, the pitch classes of the key of C stand in for scale degrees). When interested simply in pitch classes, I use the entire piece. The second corpus was comprised of music21's *Bach chorale corpus*, which includes all non-repeated chorales in the Riemenschneider edition. When tracking scale-degree usage, I use only the first four measures of the major-mode chorales transposed to C.

For analysis, the xml files' metric information was used to associate pitch information with one of five metric pulses: the measure, half-measure, beat, division, or subdivision pulse. Each pulse was determined relative to the measure length such that the "beat" would correspond to the quarter-note pulse in a measure of 4/4, and was compiled using music21's Music21Object.beatStrength parameter.

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[11] See Christopher White, *Some Statistical Properties of Tonality* (PhD diss., Yale University, 2013); and Jon B. Prince and Marl A. Schmuckler, "The Tonal-Metric Hierarchy: A Corpus Analysis," *Music Perception* 31, no. 3 (2014): 254–70.

[12] See Michael S. Cuthbert and Christopher Ariza, "music21: A Toolkit for Computer–Aided Musicology and Symbolic Music Data," in *Proceedings of the International Symposium on Music Information Retrieval* (Utrecht: ISMIR 2010), 637–42.

This method resulted in 25,977 pulses on which some pitch event occurred in the tonal corpus, and 2,125 pulses in the Bach chorale corpus.

Analysis I: More Pitch Class Change Occurs on Stronger Metric Pulses

Figure 1 shows the average change in pcs between adjacent pulses of various strengths in the tonal corpus. (Pcs were counted if their onsets occurred on a particular pulse; held-over pitches were not counted.) A clear trend is apparent: the three stronger/broader pulses retain fewer pcs than do the two weaker/quicker pulses (the differences between this group are statistically significant according to two-sided t-tests, $p < .01$; error bars show standard error).

Figure 2 quantifies pitch-class change in another way: here, each consecutive pulse in the tonal corpus was compiled, and the pitch-class overlap between each event at which a note was added or subtracted from the texture was calculated. (Here, because of the different experimental question, held-over pitches were counted.) These comparisons were divided by the metric relationship between the two consecutive pulses: pairs whose beat strength increased, decreased, and remained the same between the consecutive events. As the figure shows, consecutive events that decrease in beat strength retain significantly more pcs than do those that increase in beat strength and those that retain the same beat strength. (All averages are statistically different according to two-sided t-tests, $p < .05$; error bars show standard error.)

Figure 1. Pitch-class change on each consecutive pulse strength

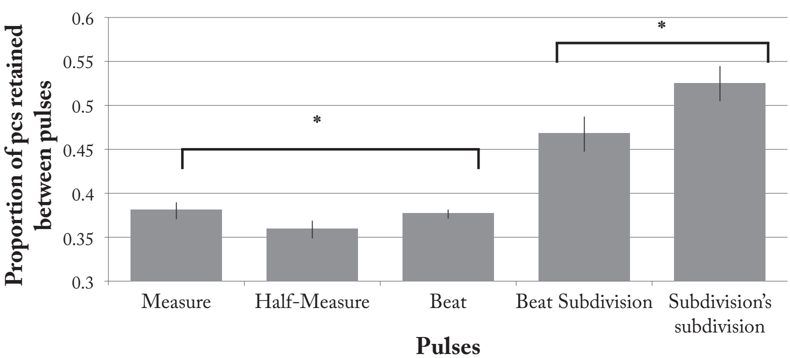
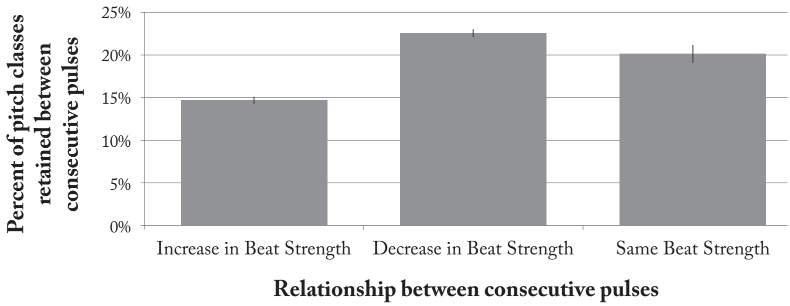


Figure 2. Average amount of pitch-class change between consecutive Pulses organized by consecutive metric strengths



In various ways, then, this corpus data indicates that music in this repertoire changes more pcs when moving from relatively weaker metric pulses to stronger pulses, and vice versa. The following analysis asks whether certain types of chords seem to be favored on pulses of different metric strengths.

Analysis II: The Different Chords that Occur on Stronger versus Weaker Pulses

Figure 3 reframes the data compiled in Figure 1, now showing what kinds of sets are created by the onsets of notes at different pulse levels. When the sets of scale degrees correspond to a traditional Roman numeral, the set is designated as such (i.e., "Do, Mi, and Sol" is designated as a I triad). In the case of non-triads, only the constituent scale degrees are shown. The figure shows that onsets on stronger pulses tend to articulate traditional triads—particularly the tonic, dominant, and subdominant triads (I, IV, and V chords)—while onsets that appear at the subdivision level tend to be diatonic singletons.

Figure 4 models the Bach chorale corpus in a similar way, now specifically tracking these three most-frequent triads. The figure shows that the stronger the pulse, the more I, IV, and V chords (C, F, and G in C major) appear on that pulse; the weaker the pulse, the less likely those triads are to appear. The trend lines associated with each data series express these opposite trends. (The "I, IV, V" regression line's equation is

FIGURE 3. Scale-degree sets formed by onsets on pulses at various metric levels

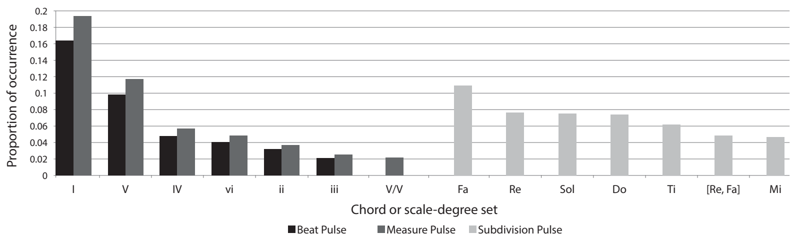
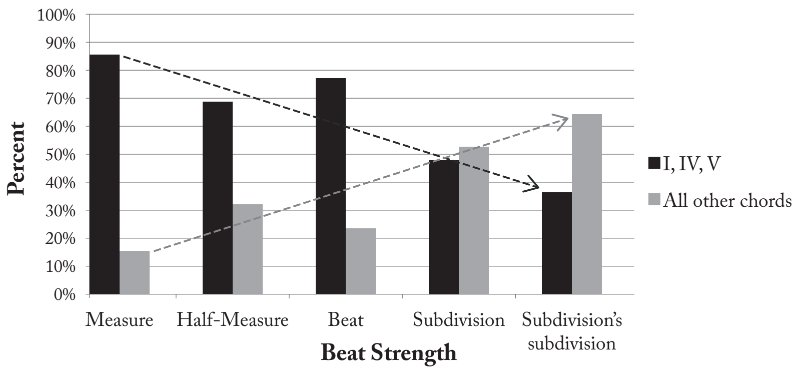


FIGURE 4. Proportion of I, IV, and V triads versus all other chords on each metric pulse in the bach chorale corpus (with trend lines)



y = –0.12x + 0.99, with r^2 = 0.85; the "All other chords" regression line's equation is y = 0.12x + 0.01, also with r^2 = 0.85.)

Discussion

Overall, these analyses show two trends within these corpora: stronger pulses change more pcs than do weaker pulses, and certain chords (in particular I, IV, and V) occur more on relatively metrically accented pulses than on unaccented pulses. I now turn my attention to the extent to which these trends account for actual harmonic practice in tonal repertoires. If we think of these trends as "rules" within a generative system, would a computational simulation of harmonic progressions that uses only these rules approximate observed harmonic practices?

The Generative Machine and Its Validation

One strength of computational research is that it can model what *might* be true, specifying all the parameters, engineering, inputs, and outputs that follow from some series of conjectures.[13] Following this sort of logic, this section describes a model that produces progressions of chords using only two rules based on the above corpus analyses. This strategy will allow us to observe what chord progressions would look like if their generative logic *only* conformed to the metric properties of harmony changes rather than to any traditional syntaxes and grammars. Creating this model will allow us to compare this metrically-generated series of chords to other more traditional models. To this end, the model's output is compared to several other corpus-based models, a baseline model, and an expert system drawn from music-theory discourse.

The Metric Model

The generative *metric model* assumes a metric grid alternating weak and strong pulses, and populates these pulses with diatonic triads. The machine approximates the two trends described above. To model the fact that greater pc change correlates to greater pulse strength, progressions from strong-to-weak pulses (or strong-to-weak transitions) change fewer pcs, while weak-to-strong pulse transitions change more pcs. Given that there are four possible pc changes between two triads (0, 1, 2, or 3 pc changes), this rule divided those four possibilities equally into two sets: weak pulses changed zero or one pcs from the previous pulse, and stronger beats changed two or three pcs. To capture the fact that different kinds of chords tend to occur on stronger versus weaker pulses, the model was parameterized such that strong pulses are twice as likely to host I, IV, and V chords than they are to host any other triad (ii, iii, vi, vii° triads), with this likelihood inverted for the weaker pulses.

Equations 1–4 formalize the general case of the metric model. Equation 1 stipulates a chord progression C with ordered timepoints 1 to n, but where each constituent c is also contained within some constrained group of possible c's, the set $\{c_0 \dots c_k\}$. The equation further notes that each c in C is itself a set containing integers. Equation 2 defines a set

[13] See David Temperley, "Computational Models of Music Cognition," in *The Psychology of Music,* 3rd ed., ed. Diana Deutsch (Amsterdam: Elsevier, 2012), 327–68.

of metric emphases to accompany each of these timepoints and defines each m to be an integer. Equation 3 then states that the probability of some chord event c at timepoint i can be derived by comparison with the chord occurring at the previous timepoint i-1 along with the relative metric emphases. If the particular metric emphasis m at a timepoint i is less than that of the previous timepoint (i.e., the chord is relatively metrically weak), the probability of the chord's occurrence is proportional to its intersection with the previous chord. If the metric emphasis is more than that of the previous timepoint, the probability is inversely proportional to that same intersection. Equation 4 then states that the probability of a chord given some metric strength is simply the proportion of times that chord occurs on that metric emphasis over all instances of that metric emphasis. Finally, Equation 5 adds a metric stipulation to calculate a final probability, stating that the probability defined in the previous equation is combined with the prior likelihood that chord c_i appears on a pulse with a particular metric strength, normalized across all possible chords $c_{0\,\ldots\,x}$.

$$\begin{gathered} C = (c_1, c_2 \ldots c_n) \\ c_i \in \{c_o \ldots c_k\} \\ C \subset \mathbb{Z} \end{gathered} \tag{1}$$

$$\begin{gathered} M = (m_1, m_2 \ldots m_n) \\ M \subset \mathbb{Z} \\ m_i \neq m_{i\text{-}1} \end{gathered} \tag{2}$$

$$P\left(c_i \mid c_{i-1}\right) \propto \begin{cases} c_i \cap c_{i-1} & m_i < m_{i-1} \\ \dfrac{1}{c_i \cap c_{i-1\,i}} & m_i > m_{i-1} \end{cases} \tag{3}$$

$$P(c_i \mid m_i) = \frac{P(c_i, m_i)}{P(m_i)} \tag{4}$$

$$P(c_i) = \frac{P(c_i \mid c_{i\text{-}1})P(c_i \mid m_i)}{\sum_{x=0}^{k}(c_x \mid c_{i\text{-}1})P(c_x \mid m_i)} \tag{5}$$

Figure 5. The metric model's transition matrix, showing the proportion in which chords in the left column move to chord in the top row

	C	**d**	**e**	**F**	**G**	**a**	**b**
C		0.08	0.17	0.25	0.25	0.17	0.08
d	0.22		0.14	0.21	0.21	0.15	0.07
e	0.29	0.06		0.18	0.24	0.16	0.08
F	0.25	0.08	0.17		0.25	0.17	0.08
G	0.37	0.04	0.21	0.13		0.16	0.08
a	0.23	0.08	0.16	0.23	0.23		0.08
b	0.32	0.03	0.18	0.11	0.22	0.14	

As described above, the current implementation constrained the possible vocabulary of chords $\{c_0 \dots c_k\}$ to to the universe of diatonic triads, only two values of m were used (strong and weak), and the I, IV, and V triads were twice as likely to occur on strong beats as any other triad. So designed, the model was run to produce 100,000 chords in C major, and a transition matrix for its output was compiled. Figure 5 shows the transition matrix produced by this method, with self-transitions ignored.

Corpus Comparisons

Four tables of diatonic/triadic chord transitions were chosen to validate the metric model, each derived from a different corpus. The first two corpora contained pieces in the Western-European classical style, the same style as the above corpus analyses, and would therefore provide the most stylistically and historically proximate data to the tested model. The first of these, the Kostka-Payne corpus, is a compendium of the Roman-numeral analyses given in the Kostka-Payne textbook's instructors' edition.[14] The original corpus is framed as transitions between all twelve possible chromatic chord roots: in the current study, only diatonic roots are used, only major-mode pieces are considered, and triads are assumed. The corpus contains 46 annotated excerpts from Bach (3), Beethoven (9), Brahms (1), Chopin (3), Grieg (1), Haydn (6), Mozart (7), Schubert (7), Schuman (5), Tchaikovsky (3), and the Tin-Pan-Alley composer Nat Ayer (1). Second, the Tymoczko corpus

[14] See Temperley, "A Statistical Analysis of Tonal Harmony."

comprises the transition tables provided in his *A Geometry of Music*;[15] the exact constituency of this corpus is not provided by the author, but includes Roman-numeral analyses of a selection of Bach, Mozart, and Beethoven pieces.[16] The dataset uses the seven diatonic Roman numerals in both major and minor modes, but in this study only the major-mode data are used.

Two American popular music datasets were also used: the DeClercq-Temperley and HookTheory corpora. These data were chosen specifically because they use a similar harmonic vocabulary as classical corpora (diatonic triads and seventh chords) but feature an ostensibly different syntax, thereby allowing for cross-stylistic comparisons. The former corpus contains a subset of songs from Rolling Stone magazine's "500 Greatest Songs of All Time," and its vocabulary and compilation are identical to the Kostka-Payne corpus.[17] The corpus contains 20 songs randomly selected from the Rolling-Stone list from each decade between the 1950's and 1990's (inclusive). The latter is derived from HookTheory.com's chord annotations of thousands of American popular songs.[18] The website makes transition probabilities between chords within its more than 18,000 annotated songs publically available. Only transitions involving two triads were considered in the current study. Because of the proprietary nature of the corpus, metadata is difficult to access; however, the earliest songs I have found in the corpus date from the 1950s (it contains, for instance, "Shake Rattle and Roll" [1954] by Big Joe Turner) and runs to the present. The corpus's distribution also includes many genres: the website's genre menu (https://www.hooktheory.com/theorytab/genres) not only contains categories like "Rock" and "Pop" but also "Video Game," "Disney," "K-pop," etc. This parameter could not be used to filter or sort the website's publically available data, but—anecdotally—search results appear to heavily favor the pop and rock categories.

[15] See Tymoczko, *A Geometry of Music*.

[16] To my mind, this corpus is best read as a generic representation of the Roman-numeral analytical techniques we teach our students in the typical contemporary American common-practice theory classroom. For a review of this corpus's peculiarities, see Julian Hook, "Review of Dmitri Tymoczko, *A Geometry of Music: Harmony and Counterpoint in the Extended Common Practice*," *Music Theory Online* 17, no. 3 (2011).

[17] See DeClercq and Temperley, "A Corpus Analysis of Rock Harmony."

[18] See Dave Carlton, "HookTheory: API documentation," accessed May, 2016, https://www.hooktheory.com/api/trends/docs.

Baseline Comparisons and a Method for Correlations

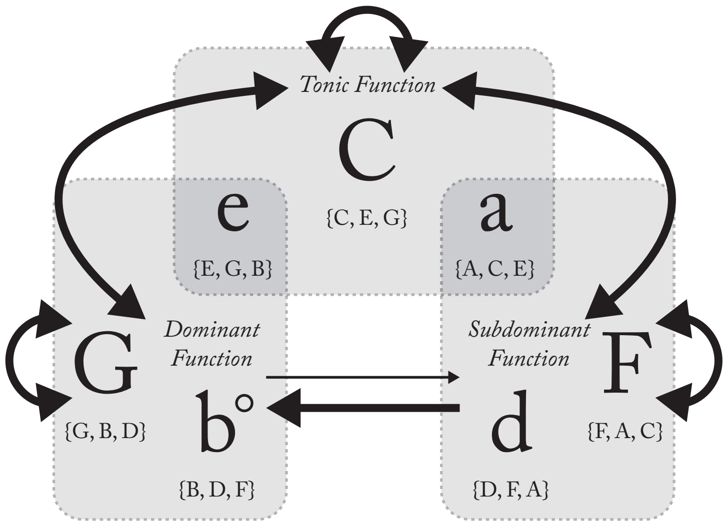
To provide some baseline comparisons, a *Frequency-only* model was created that produced triads based only on their frequency of occurrence within the Bach chorale corpus. In other words, triads were generated only by how often they occur in the Bach chorales, with no consideration of their metric position or the chords they precede or succeed. Second, a model was created using a Rule-based expert system for triadic syntax, namely an implementation of the standard Riemannian three functions (tonic, subdominant, and dominant).[19] This *Rule-based* model's grammar is represented in Figure 6, with font size showing the likelihood each chord is to be generated, and each arrow showing the probabilities for transition between chord groups. The model roughly follows the Riemannian paradigm of harmonic function,[20] and uses three syntactic categories of chords: tonic (I, vi, iii chords), subdominant (IV, ii, vi chords), and dominant (V, vii, iii chords). All categories potentially progress to each other category or to themselves, but dominant chords are twice as likely to progress to tonic as to subdominant. In the model, all categories could output any of their constituent chords, but are twice as likely to generate their eponymous primary triads as their ancillary minor/diminished triads. Again, progressions of 100,000 chords for each of these models were computationally generated; the chord progressions were used to compile the transition matrixes used in the comparative analyses.

Spearman's rank coefficient was used to correlate each transition matrix against each other matrix. This method was chosen because it returns values based on the relative ordering of chord transitions rather than the actual values associated with each (i.e., it tests whether a tonic

[19] An "expert" model is one that is based on an individual's intuitions, and in so doing reflects the knowledge of a musical "expert." This contrasts with models that set their parameters with behavioral or corpus data.

[20] Some proponents and theorists of this approach include Hugo Riemann, *Vereinfachte Harmonielehre, oder die Lehre von den tonalen Funktionen der Akkorde* (London: Augener, 1893); Daniel Harrison, *Harmonic Function in Chromatic Music: A Renewed Dualist Theory and an Account of its Precedents* (Chicago: University of Chicago Press, 1993); and Kevin J. Swinden, "When Functions Collide: Aspects of Plural Function in Chromatic Music," *Music Theory Spectrum* 27, no. 2 (2005): 249–82.

Figure 6. The Riemannian three-function model



chord moves to a dominant or subdominant triad more often, rather than *how much more* it moves to one over the other). It should also be noted that this method provides for a "randomized zero baseline"; a correlation of zero would result from a transition matrix being compared to the average completely random matrix.

Comparisons

Figure 7 shows the Spearman rank coefficients resulting from each pairwise correlation, with darker shading reflecting higher correlations. Predictably, the highest correlations result from comparisons between corpus models of the same style: note the correlations between the Tymoczko/Kostka-Payne corpora and the DeClercq-Temperley/HookTheory corpora. Also not surprisingly, the Frequency-only model produces the lowest results, although it does correlate relatively well to the Rule-based model. The Rule-based model does not correlate well to most corpus models (especially to the classical corpora), but does correlate highly to the metric model. Finally, the metric model correlates relatively well to each corpus-based model.

Figure 7. Correlations between the transition matrixes for each model

	Metric Model	Tymoczko	Kostka-Payne	DeClercq-Temperley	Hook Theory	Frequency-Only	Rule-Based
Metric Model		0.69	0.62	0.68	0.61	0.61	0.79
Tymoczko			0.84	0.68	0.62	0.42	0.53
Kostka-Payne				0.75	0.62	0.43	0.51
DeClercq-Temperley					0.81	0.52	0.72
Hook Theory						0.44	0.62
Frequency-Only							0.73

Discussion

The fact that the models derived from corpora of the same style return the highest values suggests that these methods seem to successfully track syntactic similarities. Similarly, the poor performances of the Frequency-only and Rule-based models suggest that neither simple frequency distributions nor the syntax of Figure 6 is sufficient to determine chord transitions in tonal corpora. The relatively high correlation between the Rule-based and Frequency-only model might also suggest that Figure 6's grammar does produce chords that follow tonal frequency distributions (likely because both strongly prefer generating I, IV, and V triads). However, it is also notable that the Rule-based model correlates well with the metric model. This (somewhat surprisingly!) suggests that the intuitions behind this music theoretic model might specifically correspond more to the metric/accentual properties of chord progressions rather than to actual harmonic practice.

Finally, and most notably, the metric model correlates to each corpus model with coefficients comparable to those resulting from between-style comparisons (e.g., comparing the Kostka-Payne model with the HookTheory and metric models returns the same coefficient) but less than intra-stylistic comparisons. These comparisons provocatively suggest that the metric model may be identifying some aspect of harmonic generation that is held in common between different styles. That is, the metrically-based model potentially represents a baseline preference for certain types of chord transitions that remain consistent between different historical styles, even while those styles themselves have their own syntactic idiosyncrasies.

General Discussion and Conclusions

This work shows that a model which outputs chords using only corpus-derived connections between meter and harmony reasonably simulates the chord progressions of two different tonal styles. These results 1) have implications for our understanding of traditional music theoretic models of harmonic function, 2) interact with our understanding of how chord progressions are created by composers, 3) provide a new dimension for our analyses of meter and accent, and 4) suggest several directions for future research.

Interactions with Traditional Music Theory

These findings also suggest interactions with the Riemannian three-function model. In this theory, the three functions comprise categories of chords underpinned by a prototypical triad, and—crucially—the other chords within a given function have maximal scale-degree overlap with the function's prototype. (Note that in Figure 6 each non-prototypical triad shares two pitch classes with its prototype; this property is frequently referred to in contemporary Riemannian and Neo-Riemannian discourse.[21]) This disposition cordons the seven triads into zones of shared scale degrees. However, the prototypes have minimal pc overlap with one another. This means that changes of function will entail relatively more pc change, while intra-functional moves (specifically to and from a prototype) entail relatively less pc change. Additionally, this conception of function favors the three major triads of the diatonic collection over the minor and diminished triads, framing the latter as deviations from or deformations of the former.

These tendencies parallel the metric model's treatment of strong and weak metric pulses. Just as strong pulses change more pcs, so too do changes in function; just as weak pulses retain pcs, so too do functional prolongations; just as the model prefers to metrically emphasize the diatonic major triads, so too does the functional model prioritize these structures. This similarity likely accounts for the high correlation between the metric model and the Rule-based model, but

[21] Such observations can be seen in Eytan Agmon, "Functional Harmony Revisited: A Prototype-Theoretic Approach," *Music Theory Spectrum* 17, no. 2 (1995): 196–214; Harrison, *Harmonic Function in Chromatic Music*; and Richard Cohn, *Audacious Euphony: Chromatic Harmony and the Triad's Second Nature* (New York: Oxford University Press, 2011).

also provides a provocative empirical underpinning for the intuitions behind Riemannian theory. Recalling both Lerdahl and Jackendoff's theories that harmonic prolongation should support meter and Geraint Wiggins understanding of aspects of traditional music theory as "folk psychology,"[22] this paper's modeling suggests that Riemannian theory might describe an underlying cognitive trend within metrically ordered music to produce harmonic grammars that group chords by maximal pc overlap and that distinguish these groupings via minimal pc overlap.

The Suggestion of a Generalized Property in Metric Chordal Music

While the correspondence is far from perfect, the correlations between the model's output and the popular/rock and classical corpora are higher than those of a baseline model and similar to those of between-style comparisons. The model, then, potentially captures something fundamental about the interaction of chords and meter within diatonic tonal languages. We might, therefore, speculate that composers are generally inclined to support metric accents by placing chords on relatively strong pulses that change more pcs, and might additionally prefer to align those stressed pulses with certain kinds of chords. We might further speculate that these tendencies may be held in common between different historical styles. In other words, the preference for aligning pitch-class change and the diatonic major triads with metric emphasis might provide a general inter-stylistic framework for certain types of chord progressions, with the actual instantiation of these preferences being culturally dependent. From this perspective, repertoires, traditions, and styles that rely both on harmony and meter for their musical expression might share some underlying preference in common. While provocative, this idea is simply speculative, and requires far more research and investigation.

Analyses

Music theorists' analyses of metric and rhythmic dissonance and ambiguity have generally focused primarily on patterns of loudness,

[22] Geraint A Wiggins, "The Future of (Mathematical) Music Theory," *Journal of Mathematics and Music* 6, no. 2 (2012): 2.

Figure 8. The opening piano accompaniment of "Die Rose, Die Lillie" from Schumann's *Dichterliebe*, with pc changes and eighth-note metrical accents

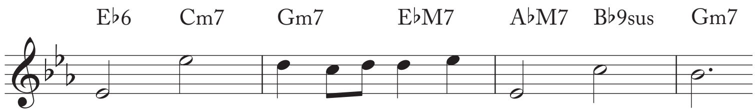
pc set:	{2,6,9}	{1,6,9}	{7,e}	{4, 7,e}	{2, 6, 9}
pc change:		1	3	2	3
Metric strength:	strongest	weak	stronger	weak	strongest

harmonic prolongation, and melodic accent.[23] This paper's findings suggest that the relationship between pc change and accent can be a valuable addition to a metric analyst's theoretical tool belt. To this end, the following four brief analytical vignettes illustrate how pitch-class and functional changes can indicate accent patterns, encourage hypermetric groupings, and even indicate a hemiola pattern that acts against a piece's expected and notated metric grid. In sum, these brief analyses provide an overview (and illustrate the usefulness) of this study's novel approach to accent.

Vignette 1. The initial measures of the piano accompaniment of "Die Rose, Die Lillie" from Robert Schumann's *Dichterliebe* (Figure 8) exhibit how pc change might be seen as expressing a metrically aligned accent. The half-note pulses are designated as the "strongest" accents, the quarter-note pulses are designated as "stronger," and the alternate eighth-note pulses are notated as "weak." The stronger and strongest pulses both align with the largest amounts of pc change, while the changes of 1 and 2 pcs occur on the weak beats. (Note: if one assumes a supertonic harmonic on the first bar's half-measure pulse, there would only be a single pc change from that harmony to the following weak

[23] See, for instance, Krebs, *Fantasy Pieces*; Peter H. Smith, "You Reap What You Sow: Some Instances of Rhythmic and Harmonic Ambiguity in Brahms," *Music Theory Spectrum* 28, no. 1 (2006): 57–97; and Mark Butler, *Unlocking the Groove: Rhythm, Meter, and Musical Design in Electronic Dance Music* (Bloomington: Indiana University Press, 2006).

Figure 9. Measure 1–4 of "Somewhere Over the Rainbow," by Harold Arlen, harmonization found in *The Real Book*[24]



pc set:	{0,3,7,t}	{0,3,7,t}	{2,5,7,t}	{2,3,7,t}	{0,3,7,8}	{0,3,5,t,8}	{2,5,7,t}
pc change:		2	1	1	2	2	3
Metric strength:	*strong*	*weak*	*strong*	*weak*	*strong*	*weak*	*strong*

beat.) While other musical parameters certainly contribute to the passage's metric and accentual structure (the return of the pattern's initial chords at bar 2's downbeat, for instance), the passage's pattern of pc change aligns with the music's metric accents.

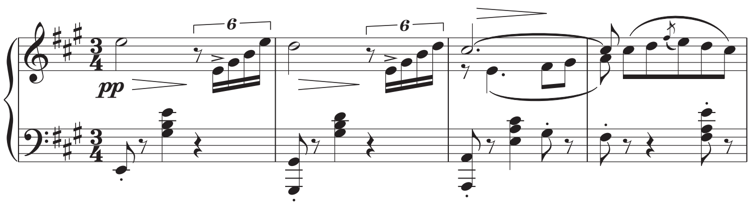
Vignette 2. The chord changes of the first four measures of "Somewhere over the Rainbow" provide another similar example. As shown in Figure 9, while not every ratio of pc change corresponds to relative metric strength (note the repeated amounts of pc change in bar 3), by and large the stronger beats host more pc change (the average pc change on weak beats is 1.2, while on strong beats it is 2.0 pcs).

Vignette 3. Figure 10 shows how this technique can be used to support a hypermeter. The figure shows the first four measures of Giacomo Puccini's aria "Quando m'en vo'" from *La Bohème.* The chords outline a similar progression to Figure 8, but are now metrically enlarged. Note also that the bars alternate between sonorities rooted in major and minor triads. Even though the piece is barred in 3/4 meter, the greater pc change and major sonorities on the alternate measures suggests a broader two-bar hypermetric pulse, with stronger emphases on measures 1 and 3 than on bars 2 and 4. Given the pc change, Puccini seems to play with the barring, hinting at a 6/4 feel superseding the notated triple meter.

Vignette 4. Figure 11 shows a yet-more subtle (and playful!) case, the end of the first phrase of *God Save the Queen.* Looking at the pc change between chords, a clear accent pattern emerges, one that outlines a hemiola (i.e., a regrouping of a 3+3 metric pattern into a 2+2+2 pattern). The pattern is, however, contradicted by other accentual signifiers: the

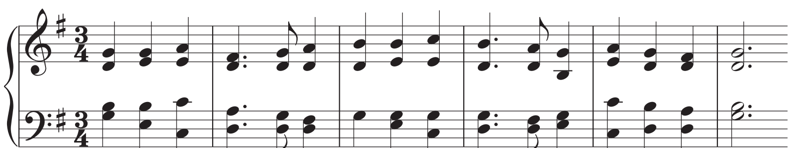
[24] See *The Real Book*, 6th ed. (Milwaukee, WI: Hal Leonard, 2004), 229.

Figure 10. PC sets and pc change in the vocal entrance of Puccini's "Quando m'en vo'"



pc set:	{4,8,e}	{3,4,8,e}	{1,4,9}	{1,6,9}
pc change:		1	2	1
Hypermetric strength:	*strong*	*weak*	*strong*	*weak*

Figure 11. PC change and hemiola patterns at the end of *God Save the Queen*'s first phrase



pc change:	3	1 3		1	3	1	3
Hemiola pattern:	<	<			<		<

parallel contours of both bars and the bass's lowest notes emphasize the barring's downbeats. If, indeed, the current model informs our sense of strong beats, it would seem that the passage invites a metrical vertigo, with a listener receiving most of their accentual information aligning with the notated downbeats, but with the pc-change accents offering alternate and contradictory accentual information. This passage shows how a composer (or in this situation, a folk tradition!) can utilize the connection between pc change, harmonic syntax, and metric emphasis to creatively play with the accentual and metrical structure of a passage.

Limitations and Future Directions

Of course, this modeling is limited and incomplete. While free composition has many interesting and complex metric grids, this paper addresses only simple strong/weak duple frameworks. Additionally, the metric and functional usage of chords like the cadential 6/4 are not directly addressed by this work and would deserve greater attention in future projects. This work also only addresses classical music and pop/rock music, and only two corpora of each; it remains to be seen whether this work applies to other corpora and to other styles. Finally, behavioral experimentation should also be undertaken to investigate whether this generative model can produce music that human listeners identify as normative, or even recognize as similar to other musical styles.

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Chris White is Assistant Professor of Music Theory at the University of Massachusetts Amherst. Chris received his PhD from Yale and has also attended Queens College–CUNY, and Oberlin College Conservatory of Music. His articles have appeared in *Music Perception*, *Music Theory Spectrum,* and *Music Theory Online* and he has presented papers at many regional, national, and international music research conferences. His research investigates algorithmic and linguistic theories of music by presenting computational models of musical style, function, meter,

and communication. Chris' research has also focused on geometrically modeling early 20th-century musics, especially the music of Alexander Scriabin and Alban Berg. Chris remains an avid organist, having studied with Haskell Thompson and James David Christie. As a member of the Three Penny Chorus and Orchestra, he has appeared on NBC's Today Show and as a quarterfinalist on America's Got Talent.