SUPPLEMENTARY MATERIAL A: THE ALGORITHM, STEP-BY-STEP

The algorithm learns a vocabulary of Roman-Numerals by observing a corpus of common-practice music and uses that vocabulary to tonally analyze some yet-unseen music.

The following therefore constructs an algorithm starts with no knowledge of Roman Numerals, triads, etc. and uses only the statistical properties of a musical corpus to undertake a key-finding task.

Overview: the steps involved to render raw statistics into a reduced vocabulary

1) Learning 1: Compile data & statistics from the raw musical surface of some corpus
2) Learning 2: Use the properties of these raw statistics to render a reduced Roman-Numeral-style harmonic vocabulary
3) Use the reduced vocabulary to analyze music

Materials: the Corpus

The Yale-Classical Archives Corpus consists of:

- 8,980 pieces by
- 505 composers with
- 12,415,229 salami-sliced (ever time a note is added or subtracted from the texture) verticalities totaling
- 3,818,712 quarter-notes of music
- Analyzed by key using music21's Bellman-Budge function (Cuthbert and Ariza 2011)
- For more information, reference White and Quinn (forthcoming) or visit ycac.yale.edu

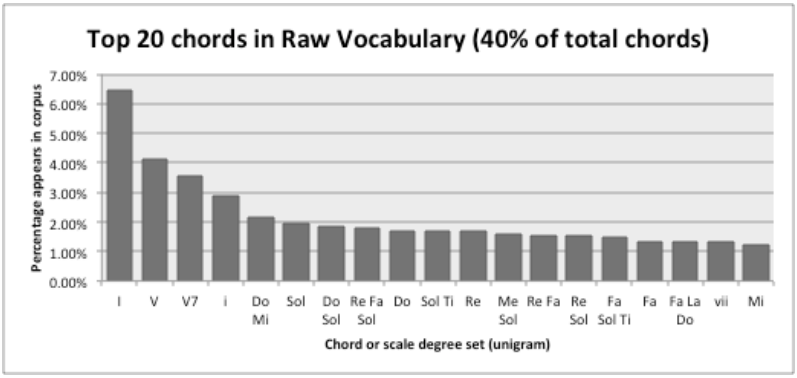
Learning 1: Compiling Raw Statistics

The learning process steps:

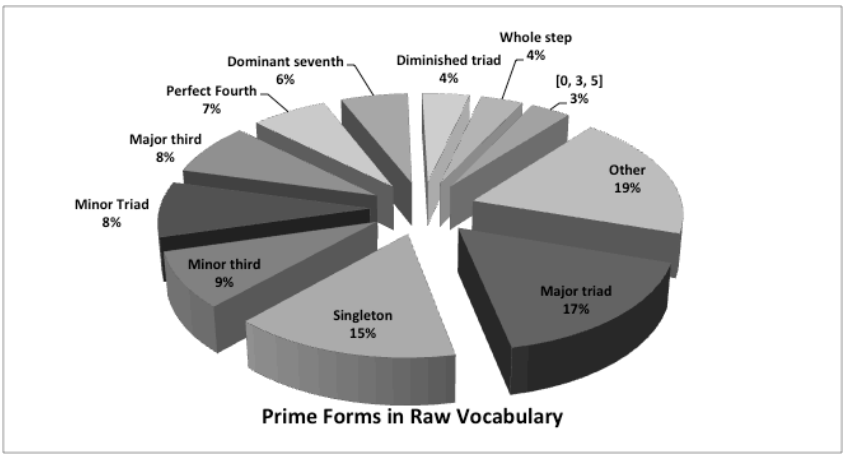
1. Remove repeats
2. If a set x of cardinality <3 is adjacent to its superset y, remove x.
3. Tally how often each verticality occurs to produce the **raw vocabulary**
4. Tally how often each succession occurs to produce the **raw transitions**

Raw Vocabulary:

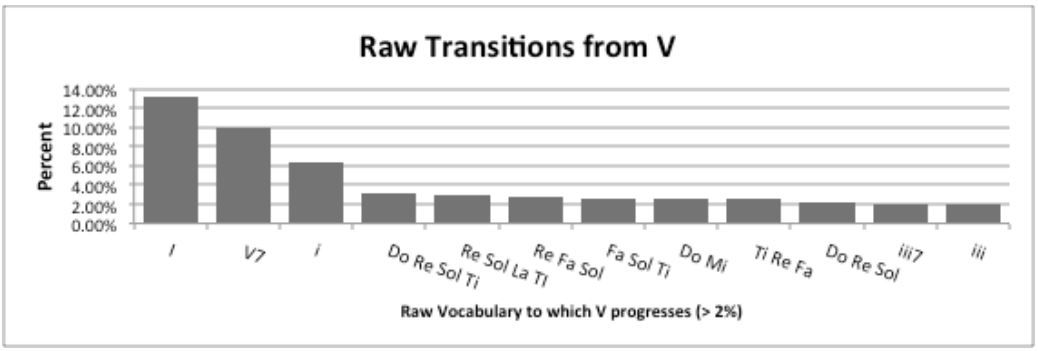
Below, the chart shows the 20 most frequent chords on the surface of the corpus. Note the differentiation of sets we would consider functionally equivalent, like V7 and <Sol Ti Fa>, or I and <Do Mi>.



The pie chart shows the prime forms present in the entire raw vocabulary. Note that the pie is very divided. For instance, all structures that appear < 3% are grouped into the "Other" slice, which occupies 19% of the distribution, the largest slice in the graph.



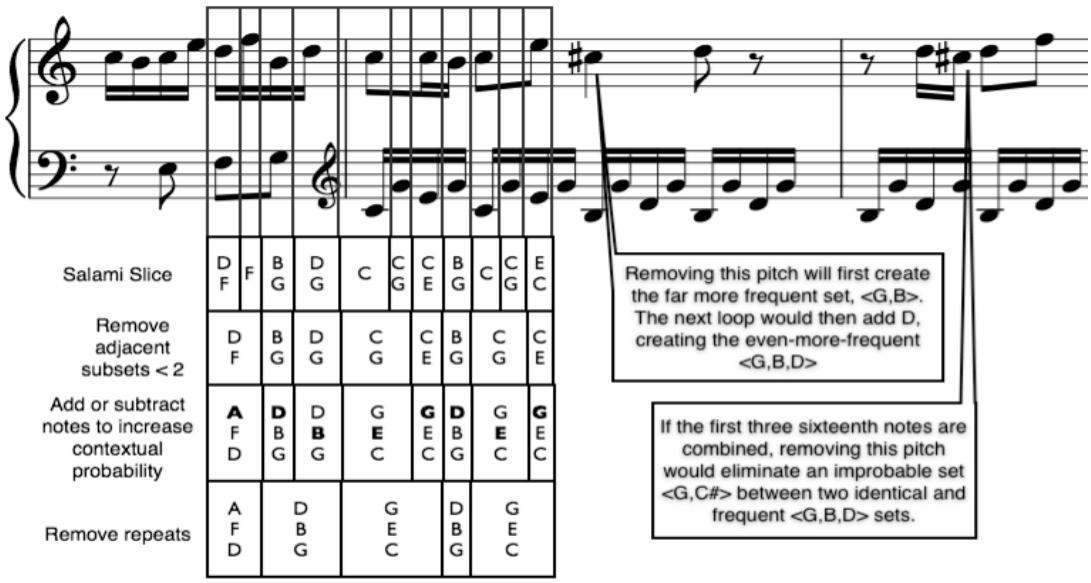
Below, the chart shows the chords to which V (or, <Sol, Ti, Re>) progresses on the raw surface. Again, note the differentiation of sets we would consider equivalent.



Learning 2: Manipulating the Raw Statistics to produce a Reduced Vocabulary

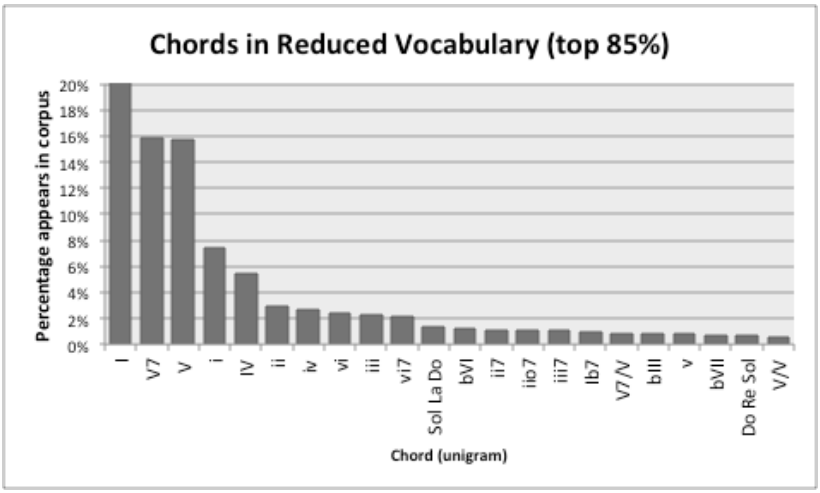
The Reduction Process steps:

1) Remove adjacent subsets
2) Remove chords not in top 85% of raw vocabulary (i.e., ignore improbable chords)
3) Remove singletons
4) Add or subtracts notes to increase the chords' contextual probability
    - Probability determined by raw transitions
    - Best solutions are determined by the Viterbi algorithm
5) Loop steps 1-4 till no further changes are made

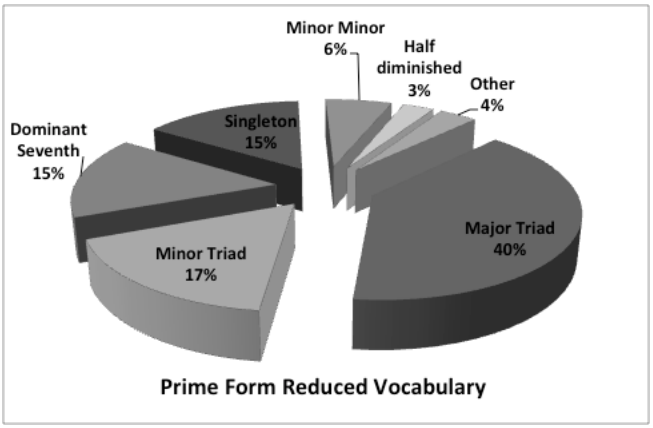


6) Tally how often each verticality occurs to produce the **reduced vocabulary**
7) Tally how often each succession occurs to produce the **reduced transitions**

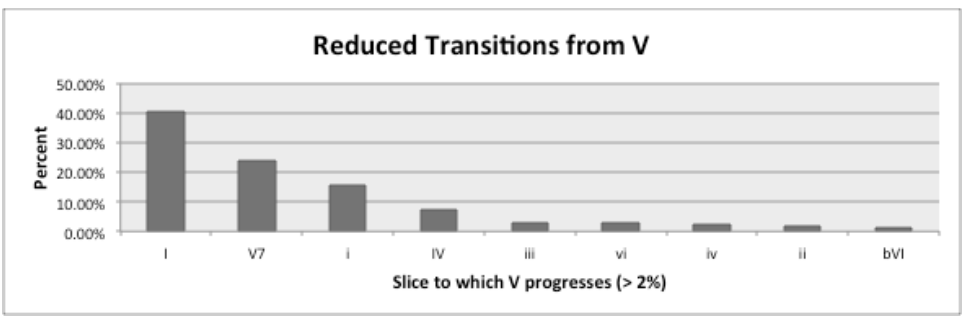
The chart below indicates the top 85% of chords in the new reduced vocabulary. Note the higher overall percentages of each vocabulary item compared to the raw vocabulary, indicating a more constrained distribution.



The pie chart indicates the prime forms within the reduced vocabulary. Note how triads and sevenths now dominate the distribution.



The graph below shows the frequencies of the chords from which V progresses in the reduced transitions. Note the more constrained distribution.



Mathematical Formalization of the Reduction Process

Formally, we can say that given a series of surface observations *O* with timepoints 1 to *n* such that…

$$O = (o_1, o_2 \dots o_n)$$

…we can reduce each *o* in *O* to a related chord *s* such that…

$$S = (s_1, s_2 \dots s_n)$$
$$s_i \cap o_i \geq 1$$

… where the intersection between each *o* and its corresponding *s* is at least 1 (i.e., they share at least one scale degree member). We produce the series *S* by maximizing the contextual probabilities *k* and the proximity of the two sets π.

$$S = \text{argmax} \prod_{i=1}^{n} P(s_i|k(o_i))\, \pi(s_i, o_i)$$

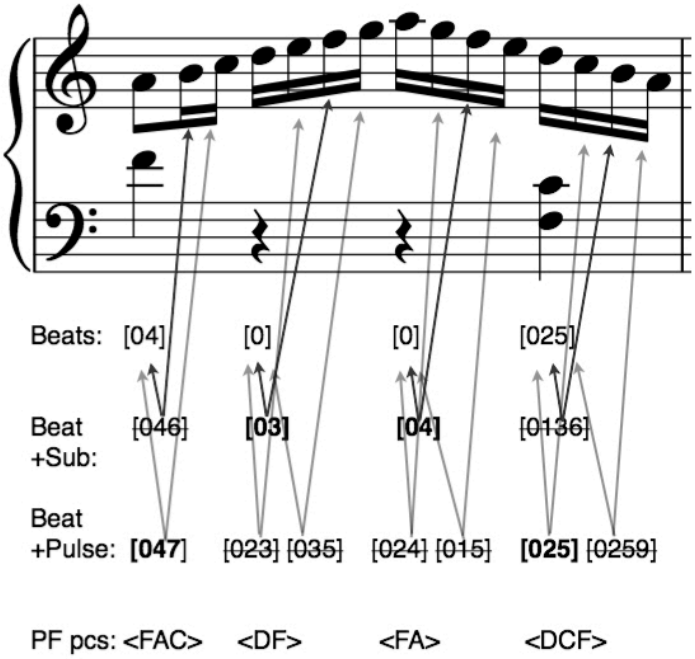
… where $P(s_i|k(o_i)$ is the probability that a given *s* would occur in the context in which we observe the corresponding *o*. In our case, this is a simple bigram probability (i.e., the

likelihood of seeing a chord given the chord that came right before it). Here, $\pi(s_i, o_i)$ is the proximity between the two sets, s and o. In our case, π is equal to the overlap (or intersection) between the two sets.

Analysis 1: Conform to Prime Forms of the Reduced Vocabulary

The steps:

1. Divide the music into three **metric level**s: the **beat, subdivision, and pulse.**
   - Zikanov 2014 is used to find recurrent periodicities in numbers of note onsets
2. List the prime forms that occur directly on each **beat**
3. For each beat, add the pcs of the next **subdivision;** list that prime form
4. For each beat & for each pulse surrounding the next subdivision: add the pcs of the next **pulses;** list those prime forms
5. If the process has found one or more prime forms present in the **vocabulary**, add the pitches of that prime form occurring most frequently in the vocabulary.



Otherwise, record only those pcs occurring on the **beat**

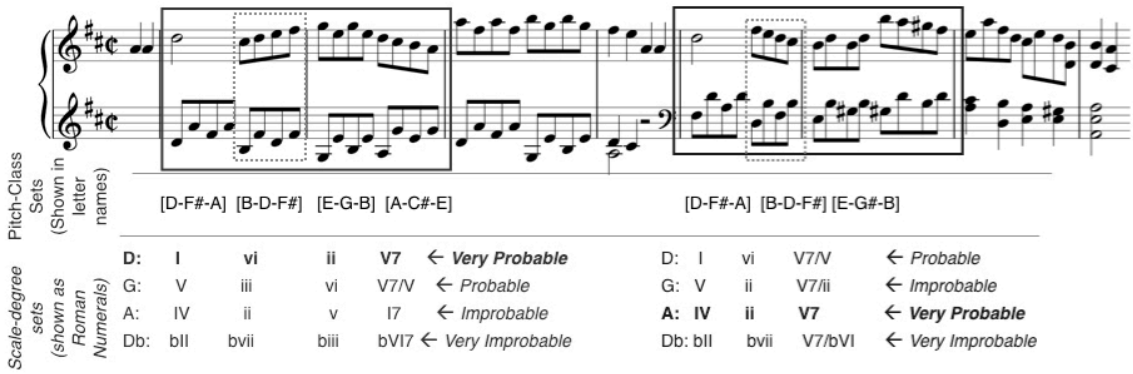
Analysis 2: Key Finding and Conforming to Reduced Vocabulary

Steps involved:

1. Select a window of music to analyze
2. For each possible key transposition of the window, proceed through the reduction process (see example in **Learning 2**):
   a. Remove adjacent subsets
   b. Removes chords not in top 85% of reduced vocabulary
   c. Add or subtracts notes to increase the chords' contextual probability
   d. Find best solution using Viterbi algorithm
3. Choose the key that assigns the window the most frequent (i.e., most probable) chord progressions

In its current incarnation, the program creates a window *x* beats long from each beat's time point. If the reduction process results in the number of chords falling below *y*, the following beat is added to the window. *x* and *y* are variable, with *y* representing the minimum length one expects the music to be in a single key.

Sample Analysis:



Things of note arising from discarded models:

- Reducing V7 to V (i.e., considering sevenths as subservient to triads in the vocabulary) creates models in which I-V and IV-I are often confused for one another
    - Highlights the importance the unique tritone of V7 in key-finding
- Not reducing to probable normal forms using meter often produces wildly inaccurate analyses, as the model confuses dissonances for chord tones
    - Highlights the connection between dissonance-consonance and improbable-probable events
    - Highlights the importance of triads and sevenths as structures separate from their scale-degree instantiations
- Not removing improbable chords from the vocabulary (i.e., taking 100% of chords rather than the 85% most probable, as reported above) creates analyses that are prone to error
    - Providing more vocabulary choices allows for more key ambiguity
- Admitting singletons into the vocabulary creates error-prone analyses
    - Highlights how ambiguous small-cardinality sets can be in the key-finding task

SUPPLEMENTARY MATERIAL B:
THE INSTRUCTION PAGE AND RESULTING HUMAN AND COMPUTER ANALYSES OF THE KOSTKA-PAYNE EXCERPTS

INSTRUCTION PAGE

Name _____ Instrument _____

Age _____

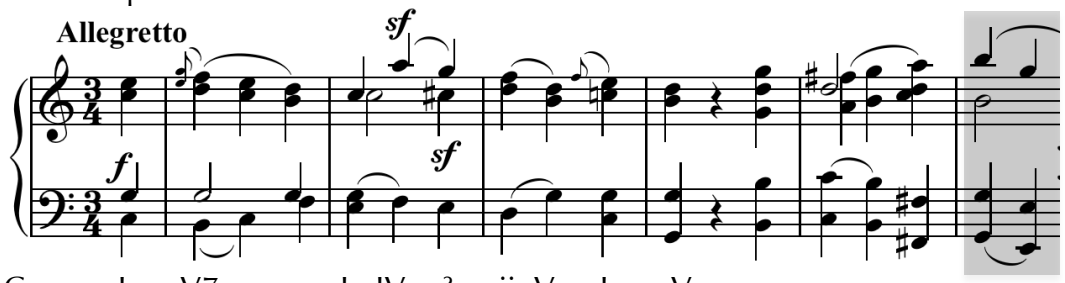
Status: Freshman Sophomore Junior Senior Graduate Student

Gender Identity: Male Female GenderQueer or Neither Prefer not to respond

Analysis:

- Write Roman numerals under the music.
- Use the fewest possible RNs, approximately 1-2 per bar.
- Show longer prolongations by writing a single chord with a line following.
- For modulations, you write the numerals of the two different keys above one another.
- For chords you don't know, write a question mark ("**?**")
- Note sevenths ($V \neq V^7$).
- USE NO INVERSIONS (V^{43} or V^{65} will just be V^7).
- Do NOT analyze the greyed-out music, but you may take it into consideration when deciding on keys or modulations.

An example:

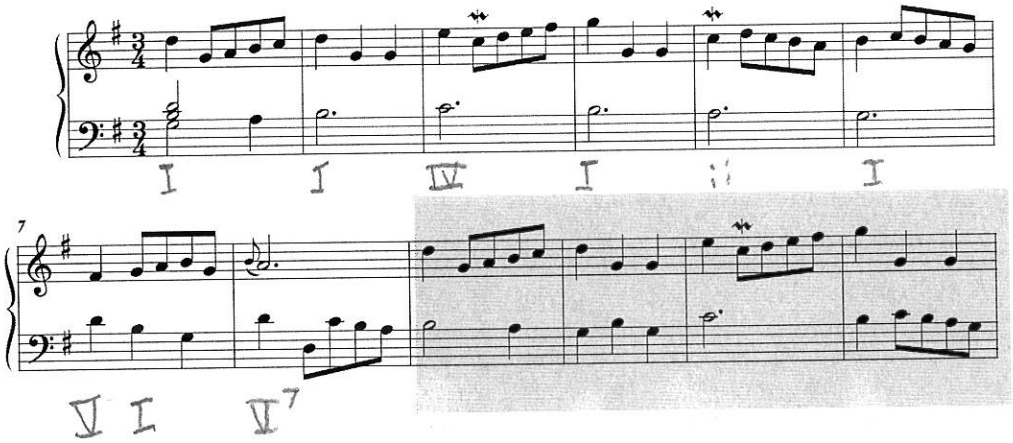


C: I V7-------- I IV ? ii V I V --------

G: I ------- V7 --------

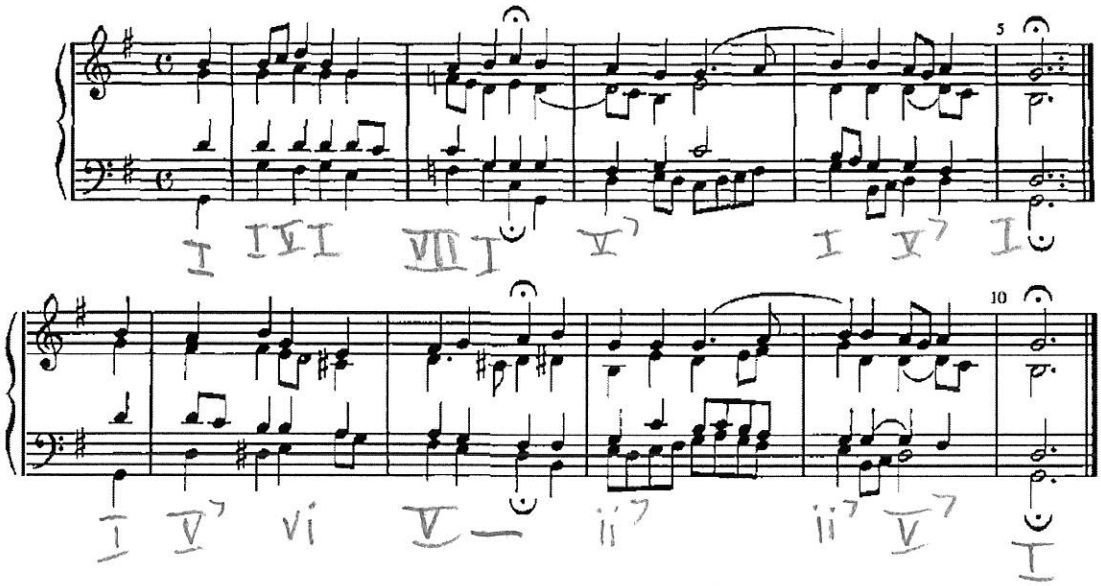
The following 32 analyses were created by sophomore music majors at the University of North Carolina at Greensboro's School of Music, Theater and Dance. The numbers below each example were randomly assigned to keep the identities of the students anonymous and to ensure their segregation from computer-generated examples.

Ex 1a, Anonymous (but often attributed to Bach), Minuet in G



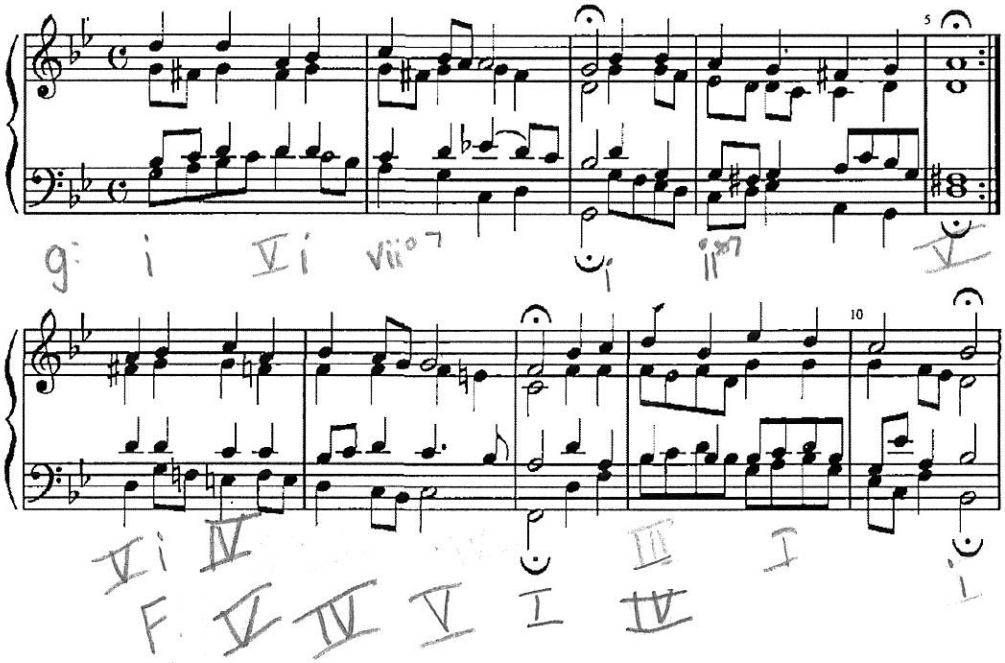
7421

2a Bach, Chorale, "Uns ist ein Kindlein heut' geboren"



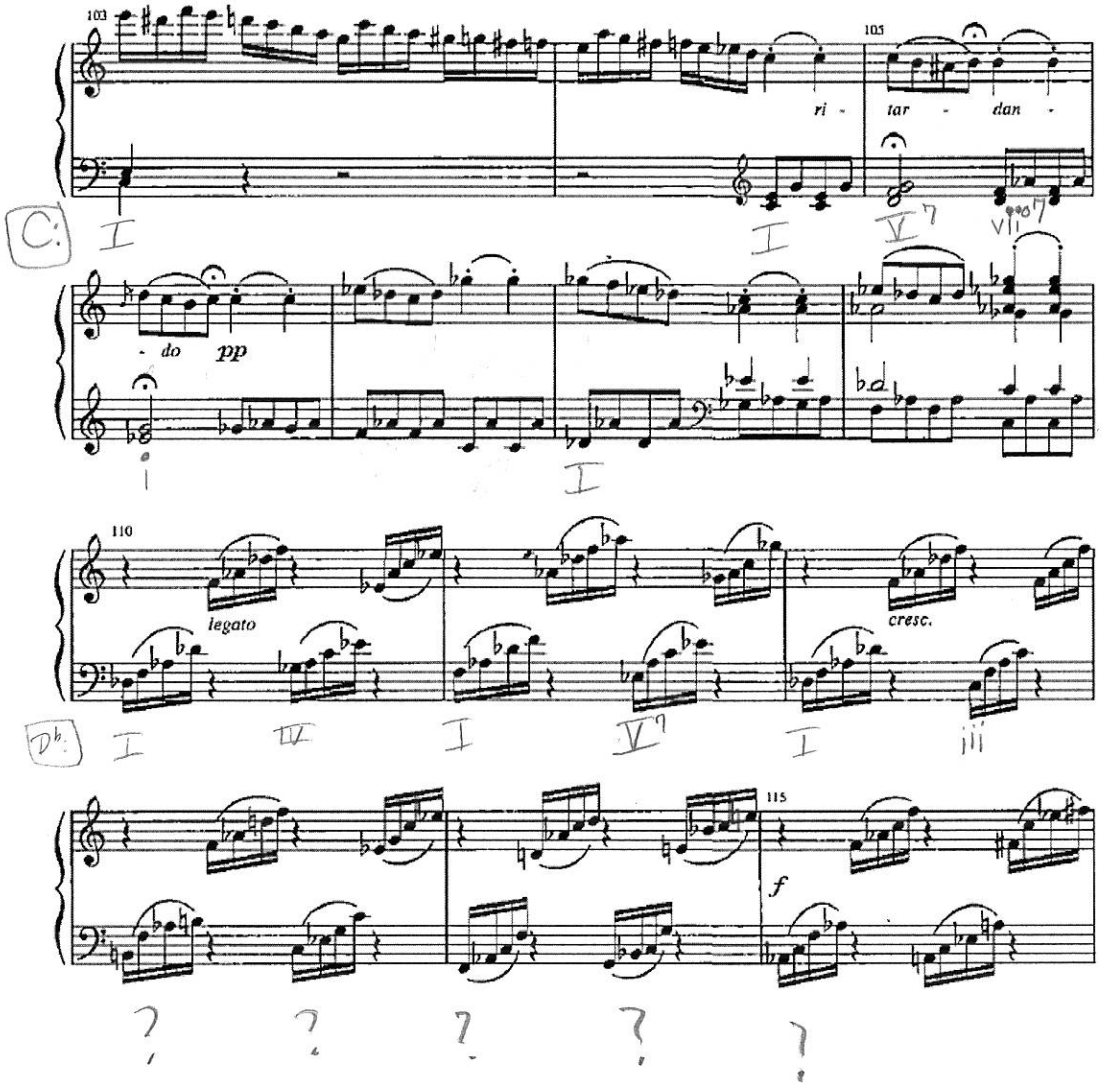
4309

3a: Bach, "Jesu, der du meine Seele"



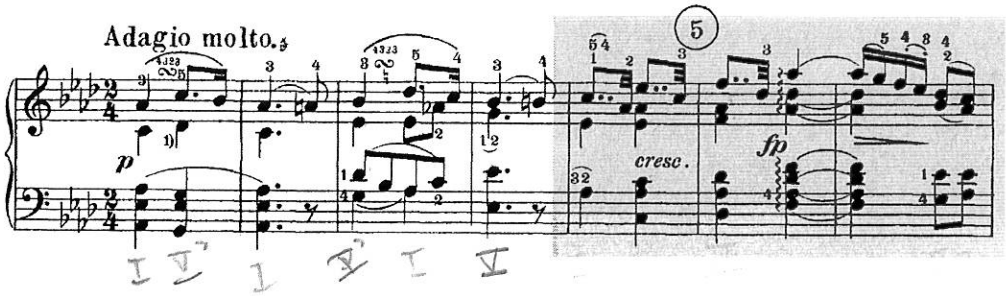
5334

4a: Beethoven, Rondo op. 51, no. 1



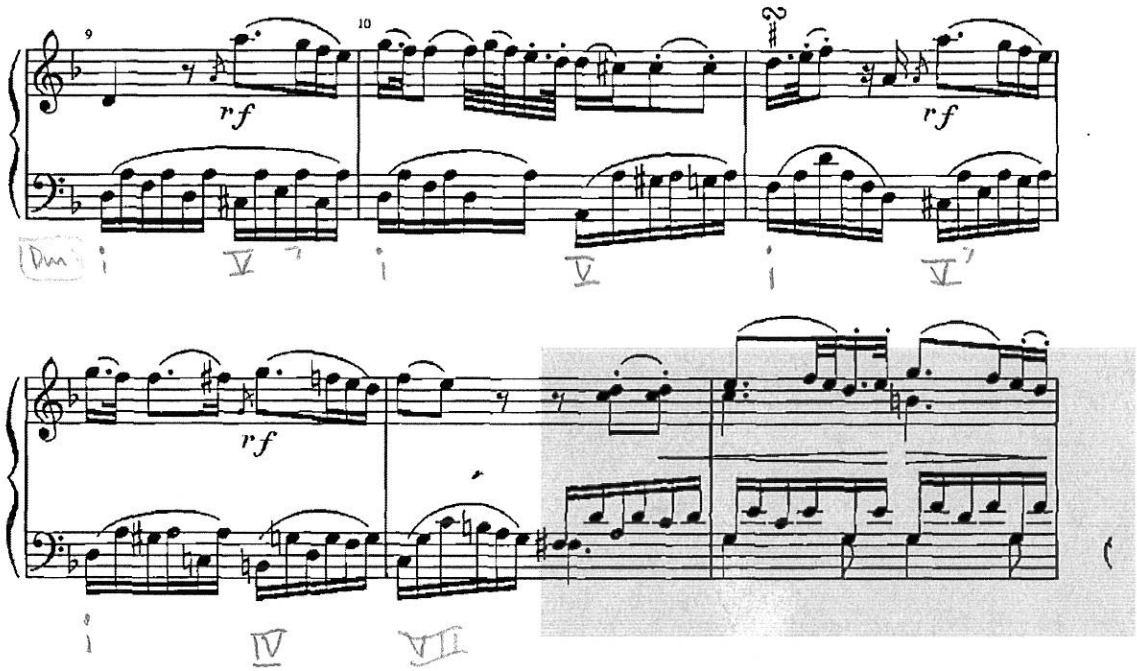
7976

5a: Beethoven, Sonata Op. 10 No. 1, ii



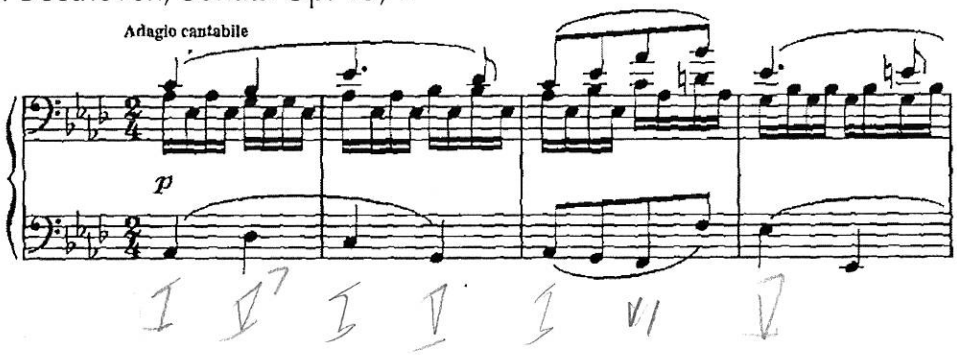
4913

6a: Beethoven, Piano Sonata op. 10, no. 3, ii



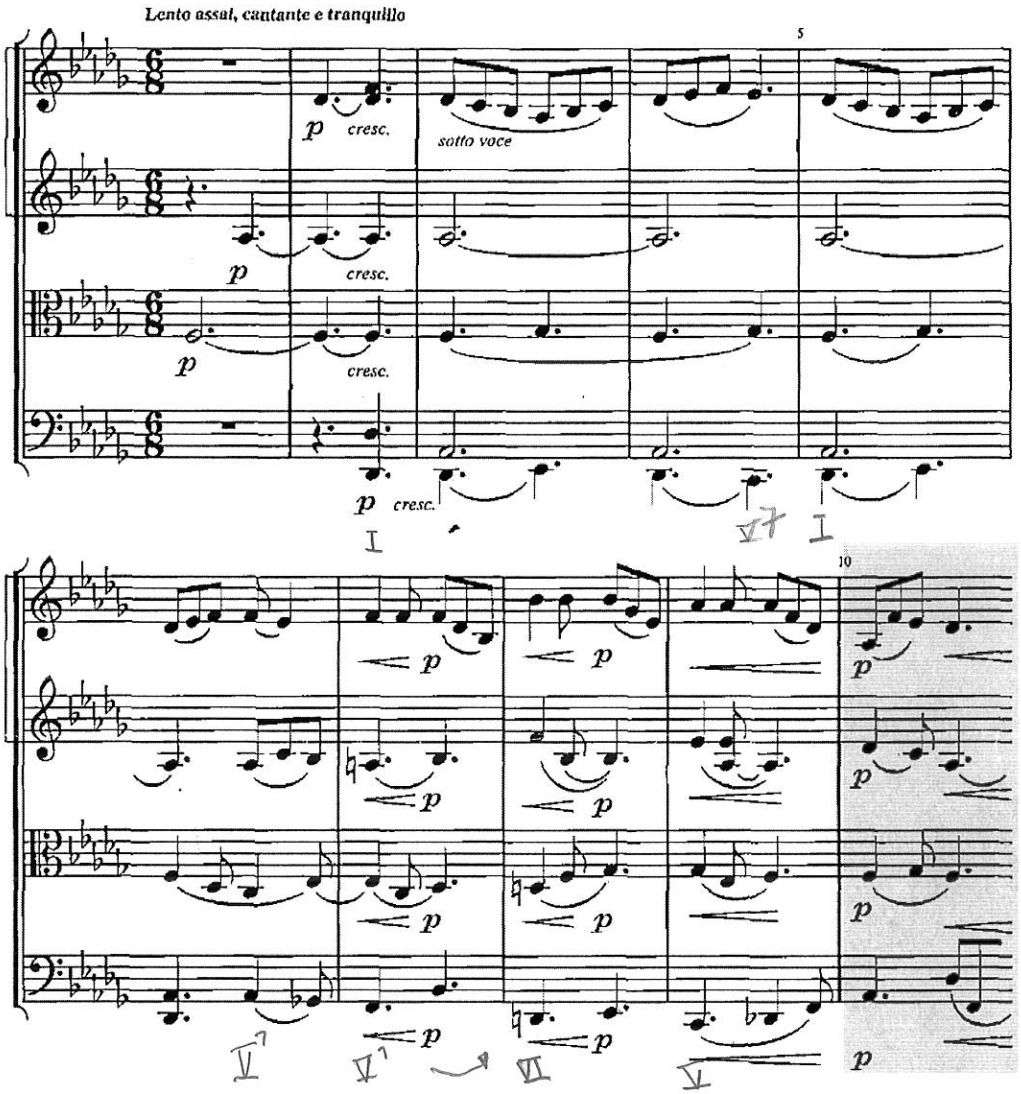
4025

7a: Beethoven, Sonata Op. 13, II



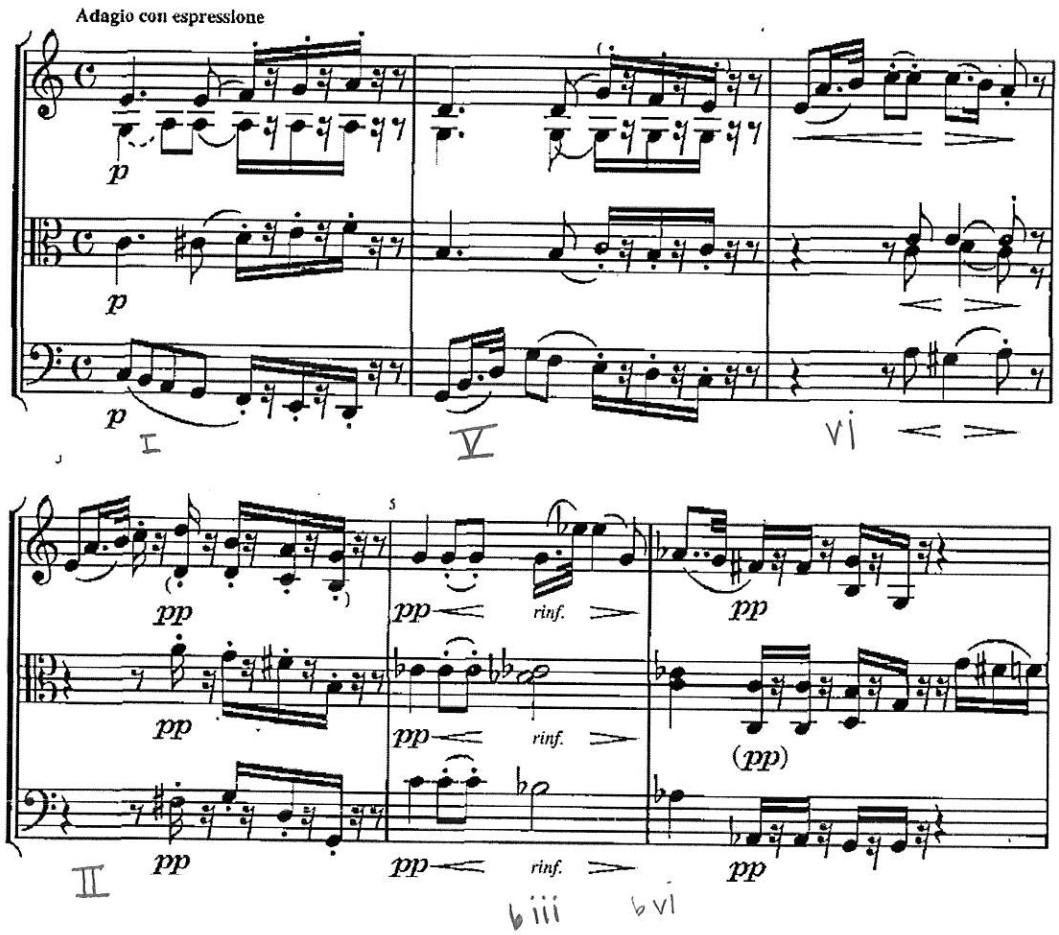
9221

11a: Beethoven, String Quartet op. 135, iii



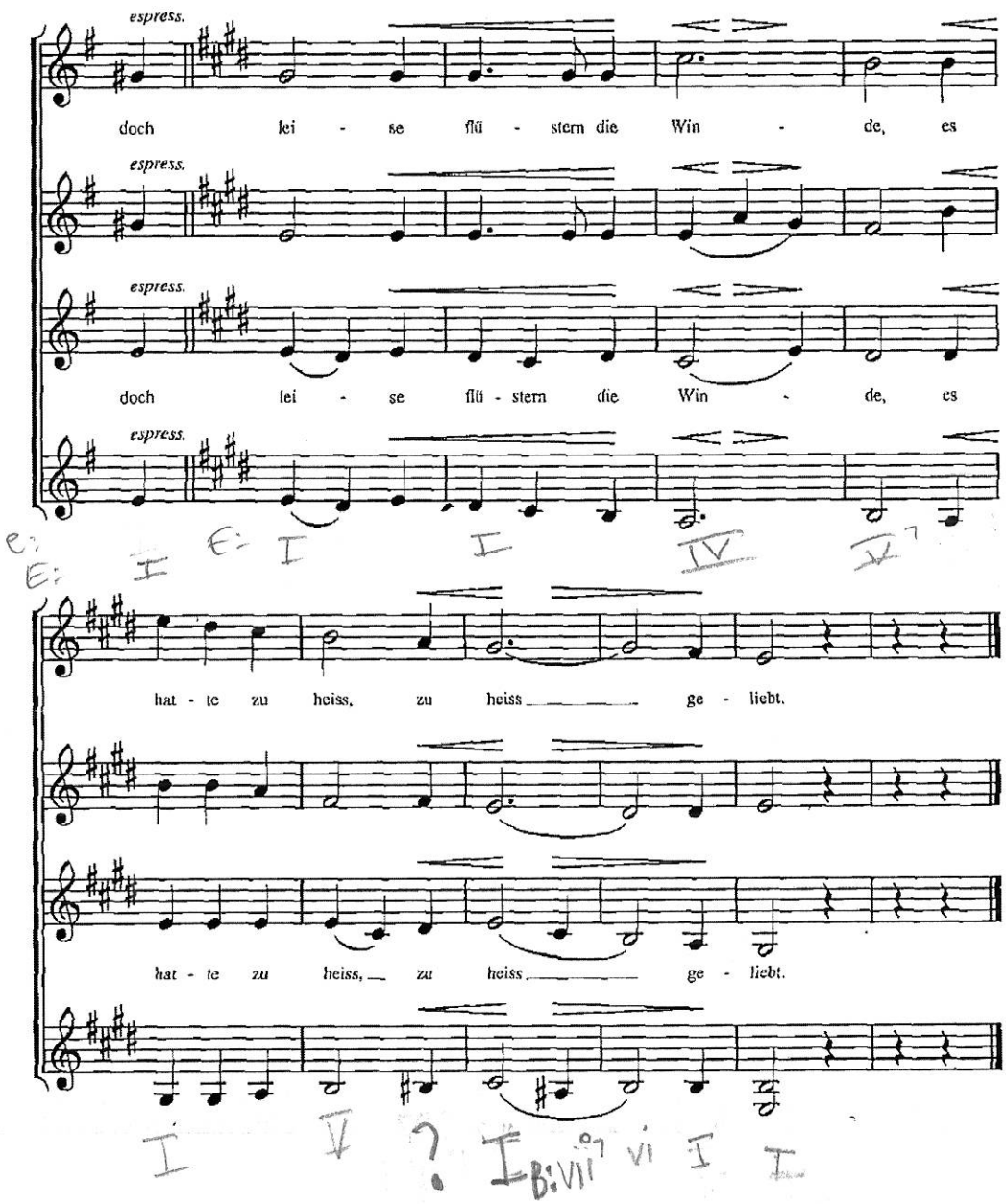
1082

12a: Beethoven String Trio op. 9, no. 3, ii



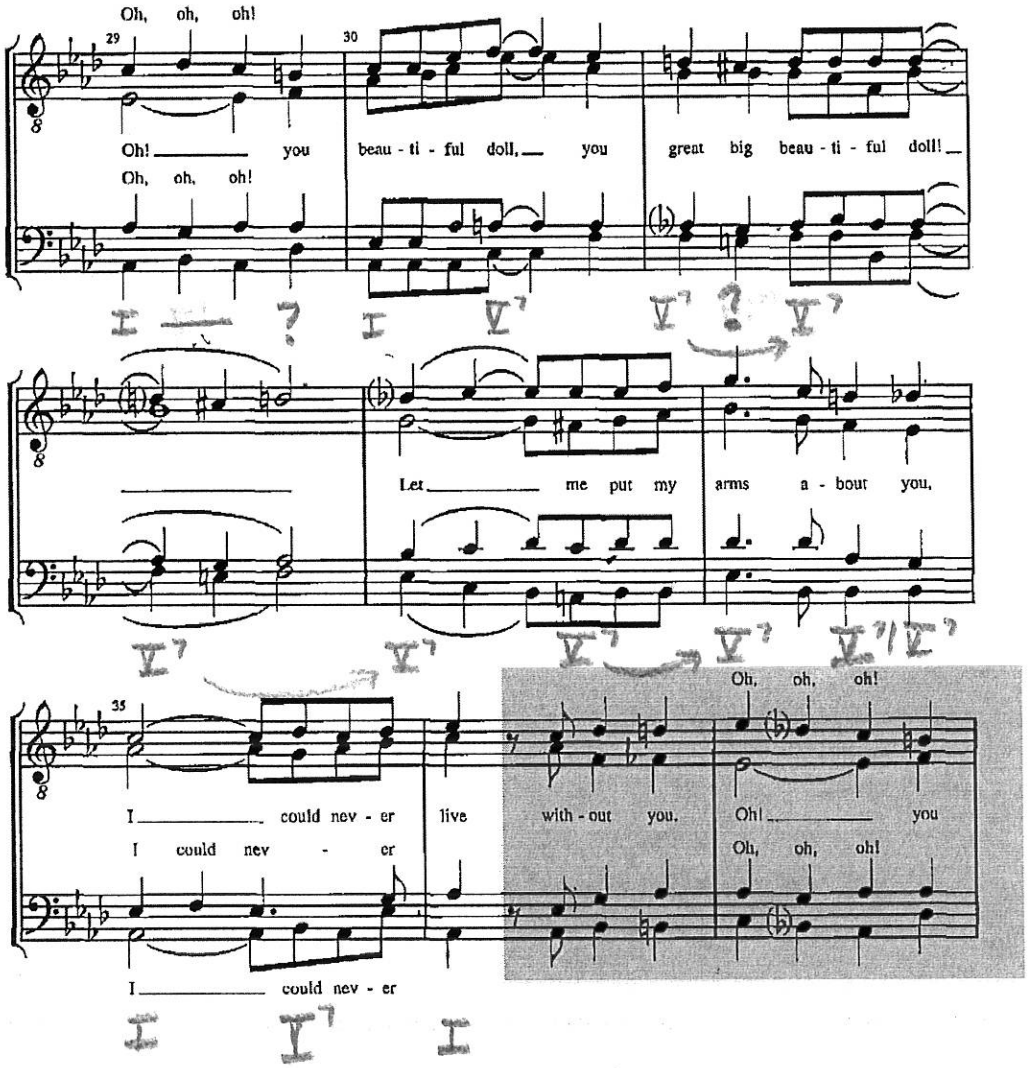
2919

13a: Brahms, "Und gehst du ueber den Kirchhof", op. 44



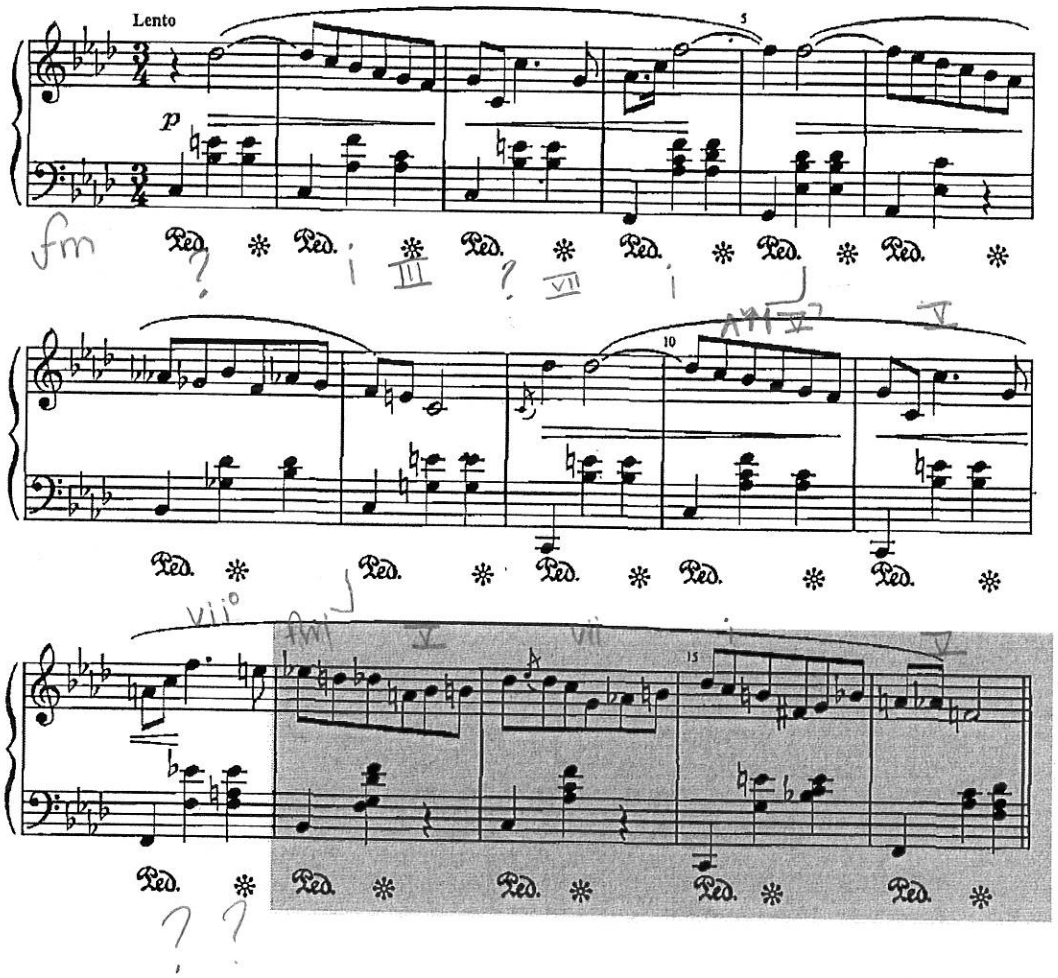
2773

14a: Ayer (arr. by Campbell), "Oh! You Beautiful Doll"



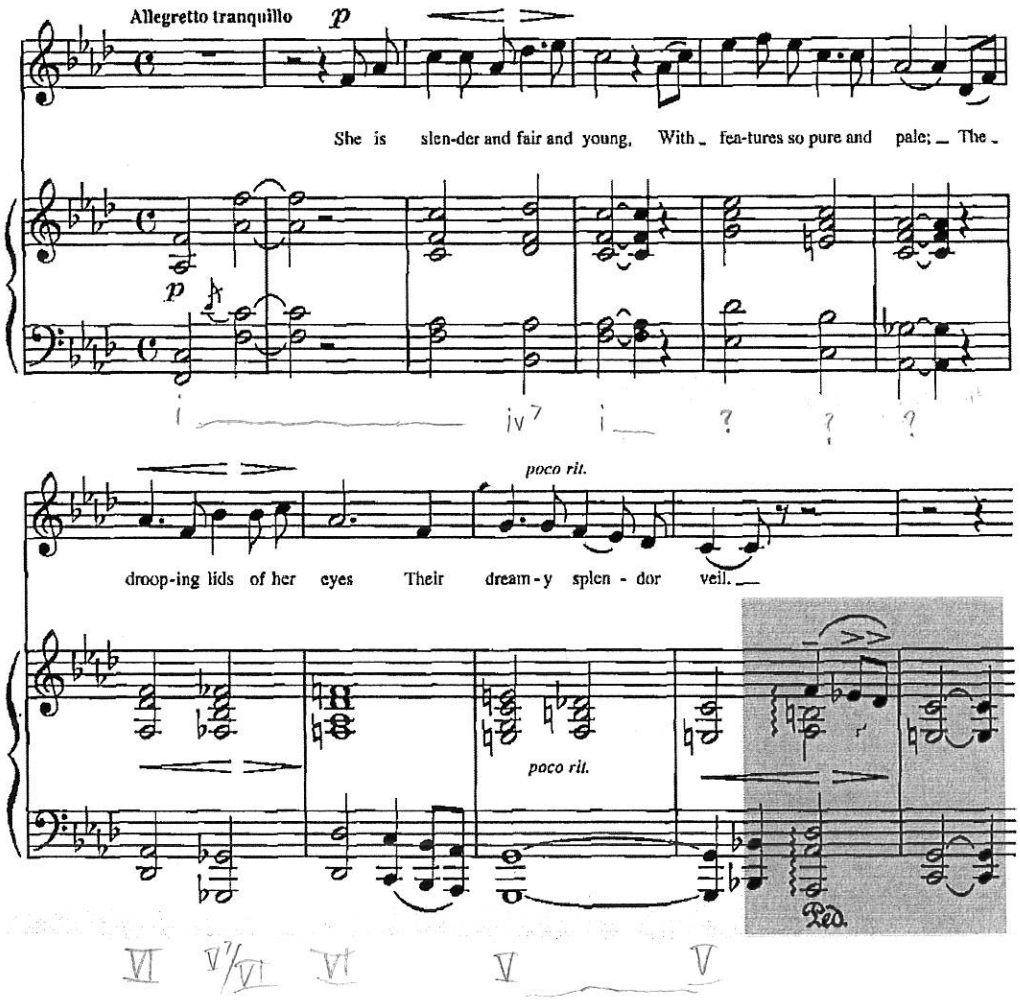
8127

16a: Chopin, Mazurka op. 63, no. 2

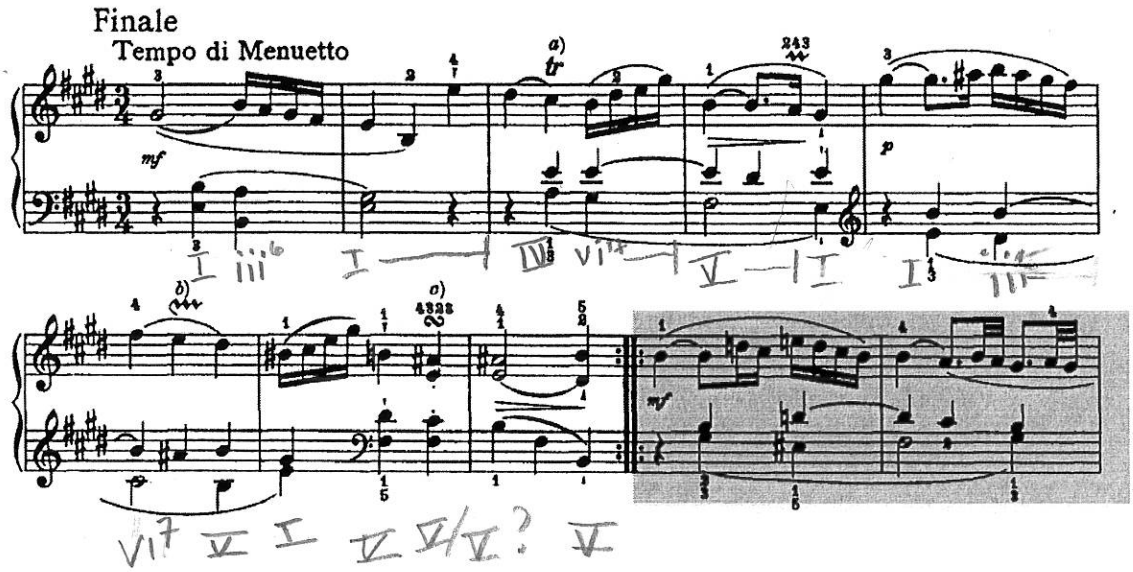


5864

18a: Grieg, "The Mountain Maid," op. 67, no. 2

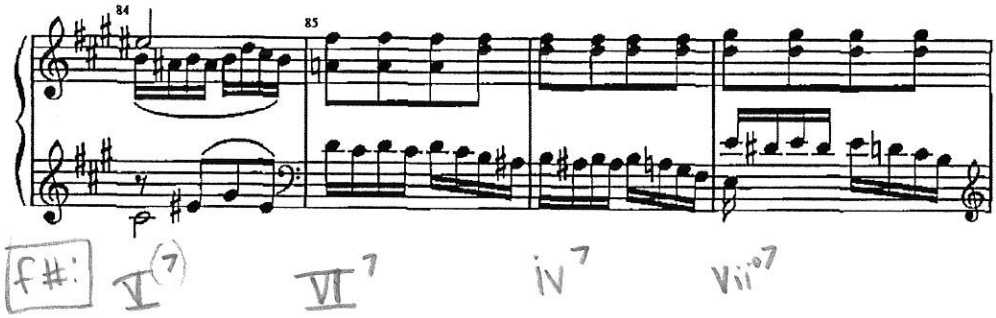


Ex19a: Haydn, Sonata No. 22, iii



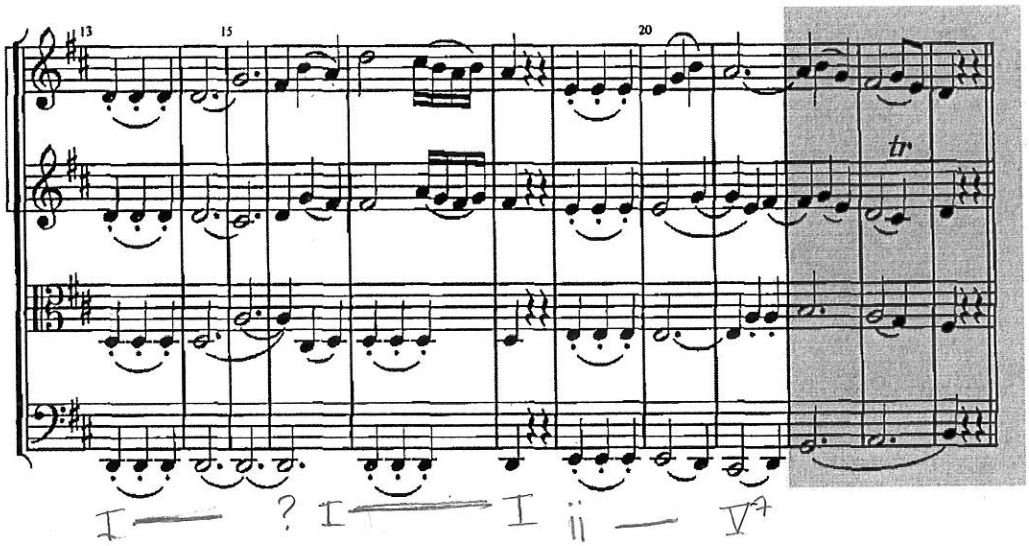
2030

20a: Haydn, Piano Sonata no 30, I



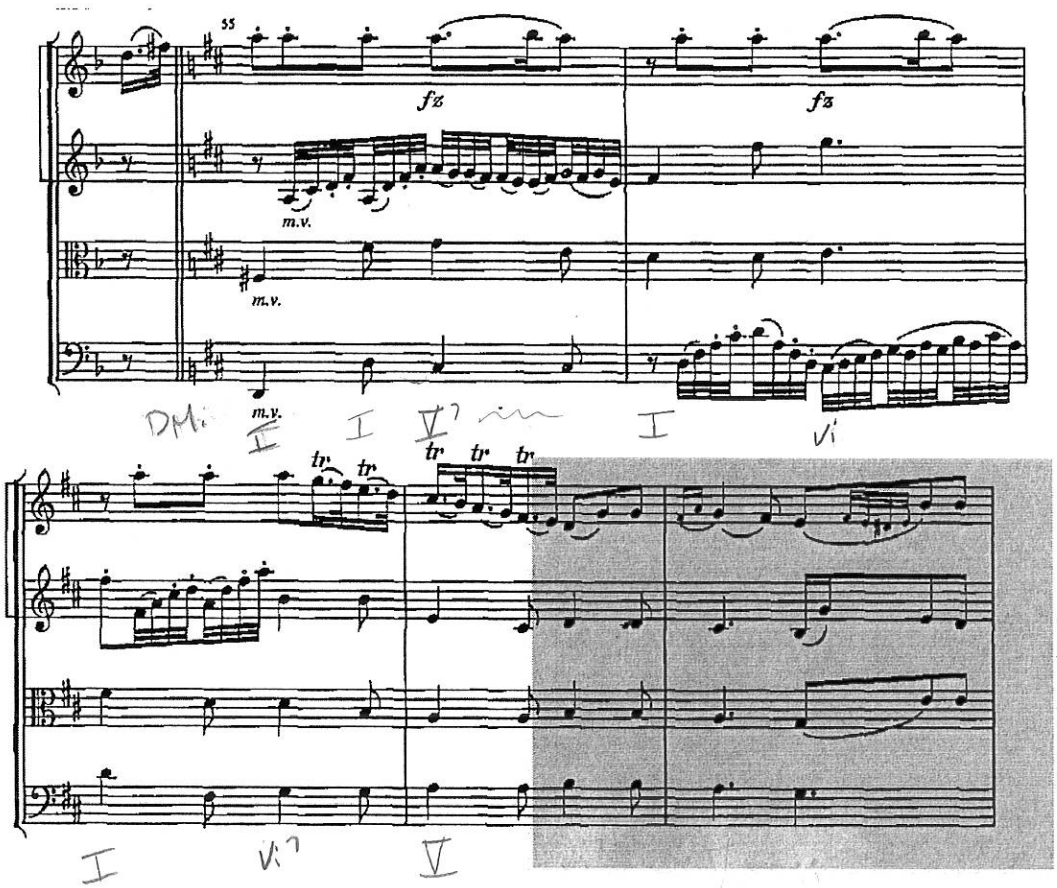
0626

21a: Haydn, String Quartet op. 20, no. 4, i

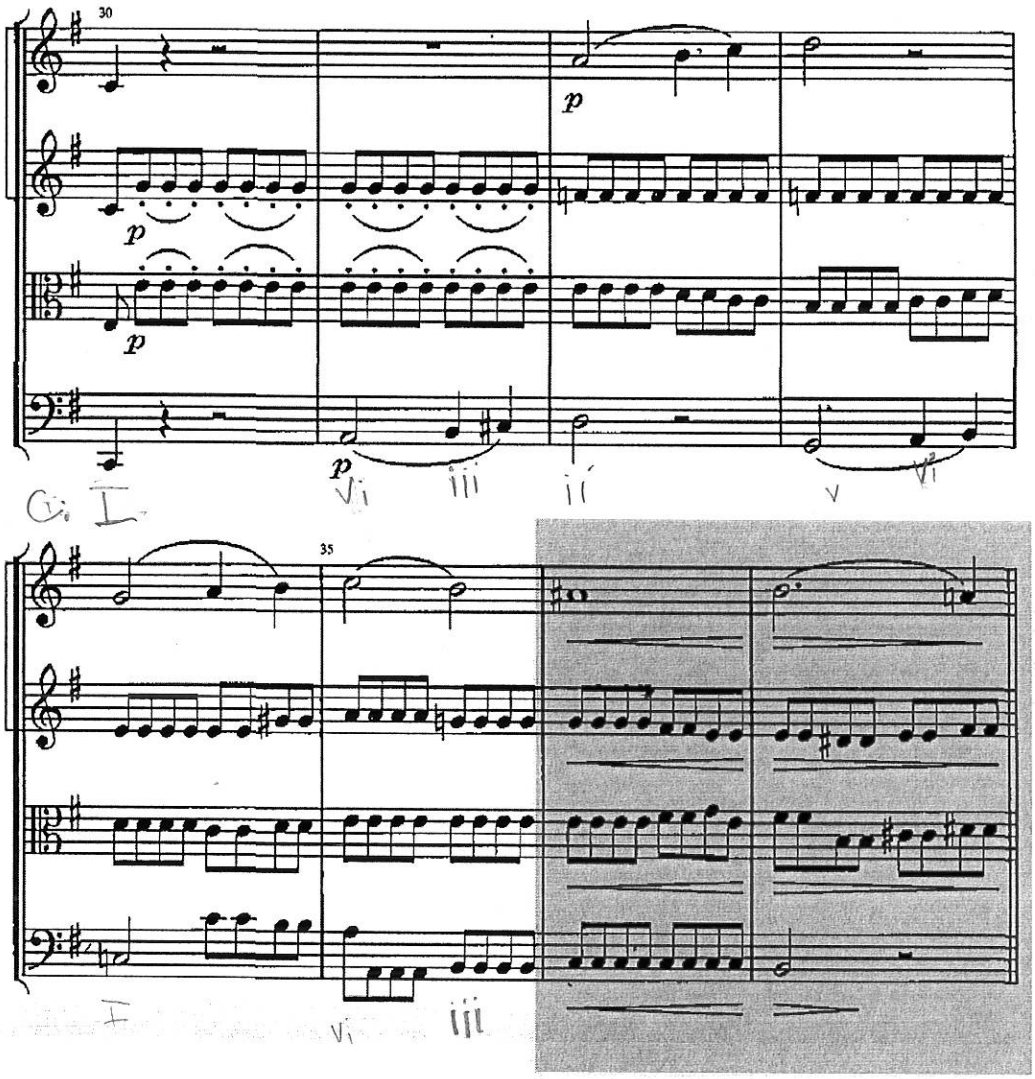


5338

22a: Haydn, String Quartet op 50, no 6 ii

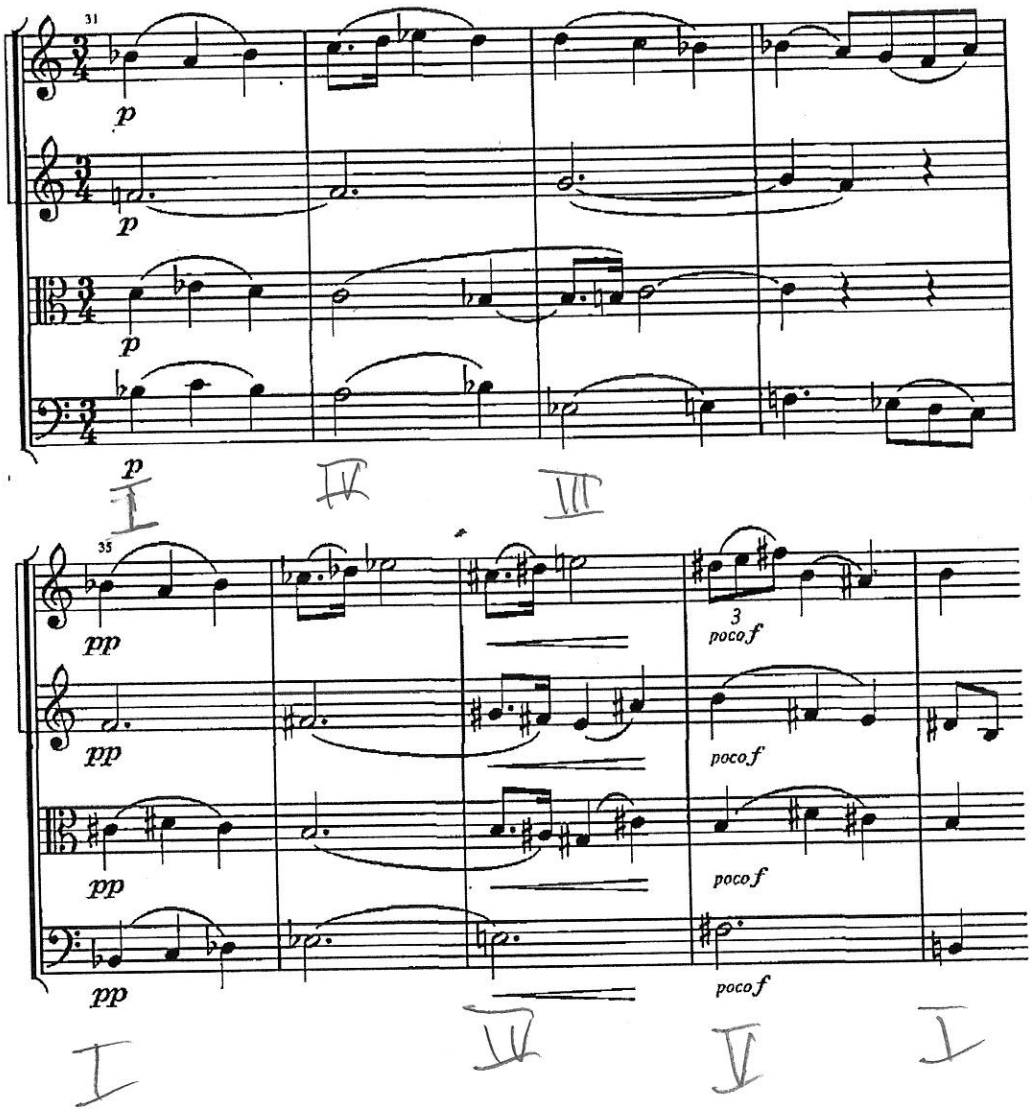


23a: Haydn, String Quartet op. 74, no 3, ii



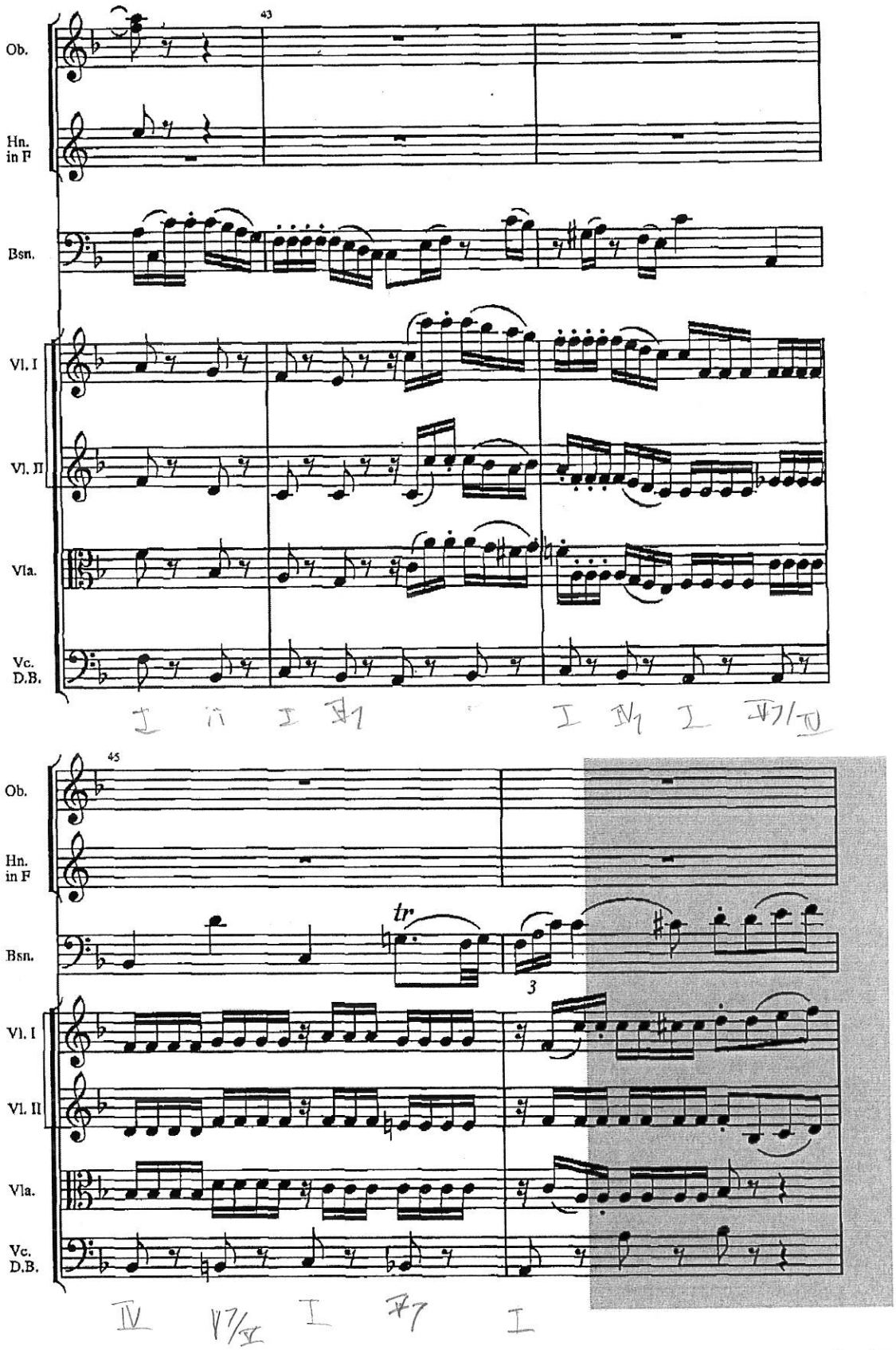
9574

24a: Haydn, String Quartet op. 76, no. 6, ii



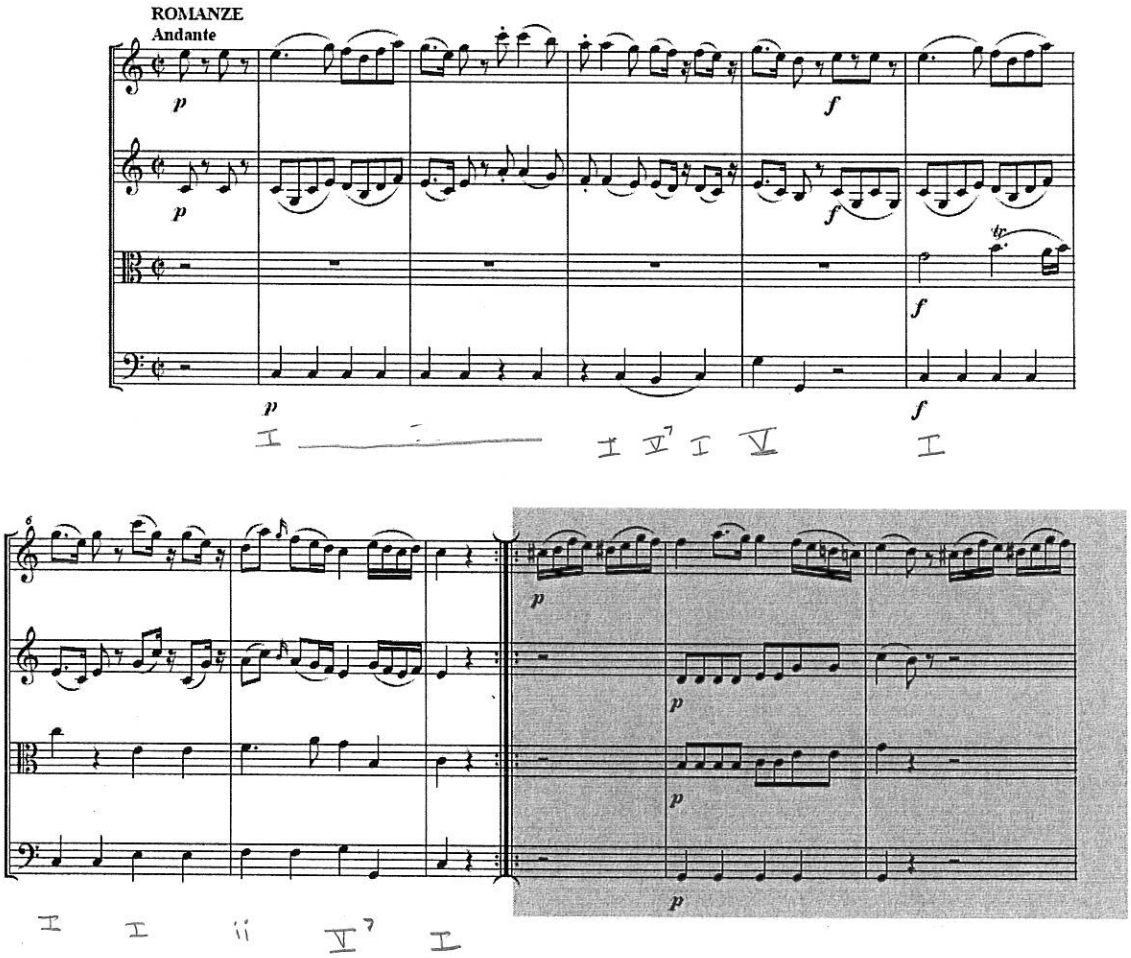
1470

25a: Mozart, Bassoon Concerto K. 191, ii



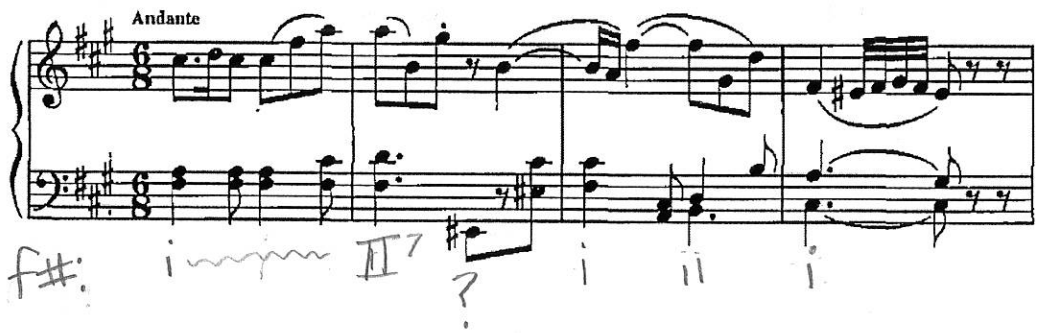
3.149

Ex 26a. Mozart, "Eine Kleine Nachtmusik", K. 525, ii



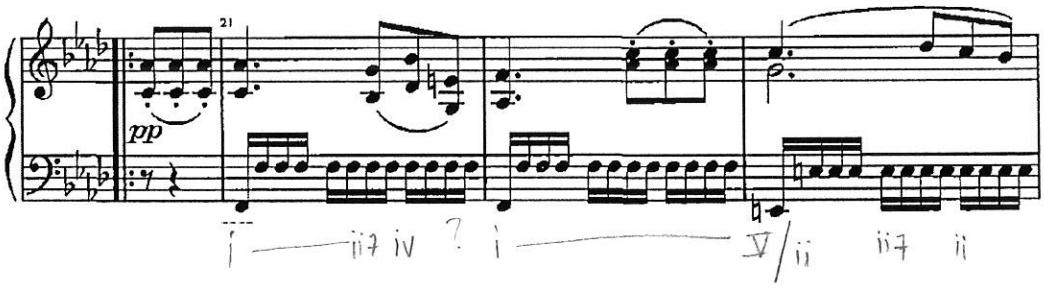
8325

27a: Mozart, Piano Concerto K. 488, ii



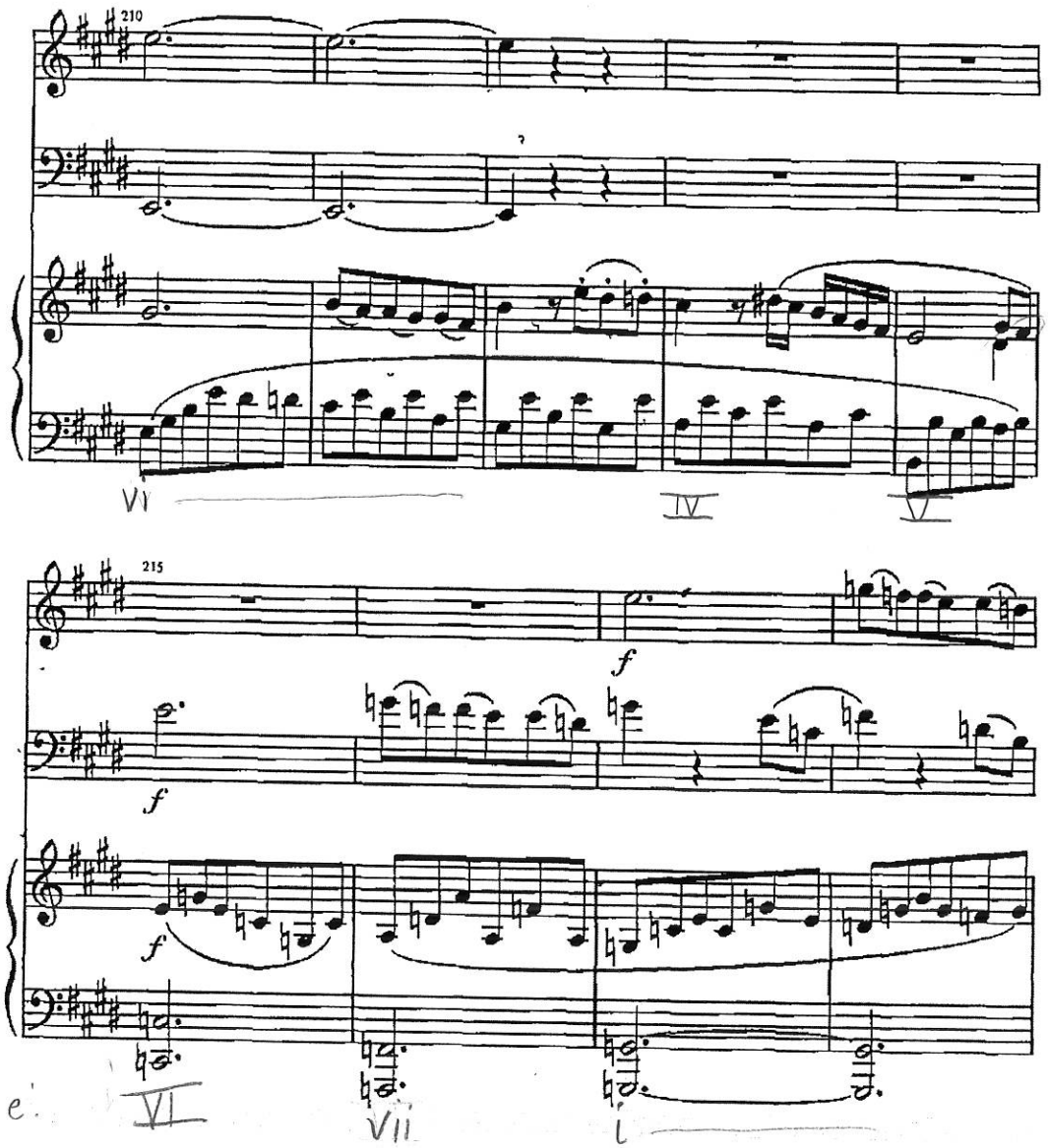
1650

28a: Mozart, Piano Sonata K. 330, ii



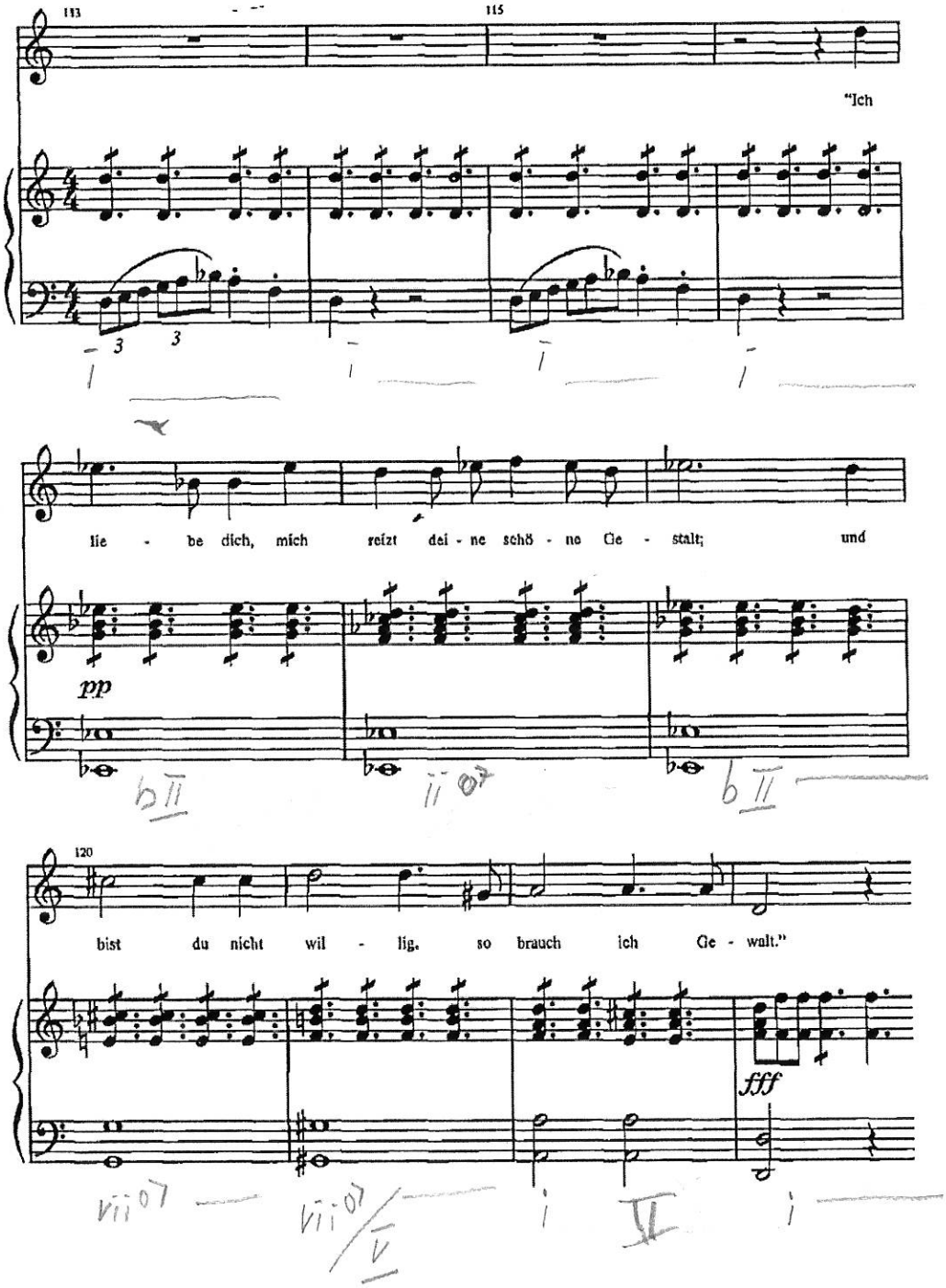
2720

30a: Mozart, Piano Trio K. 542, i



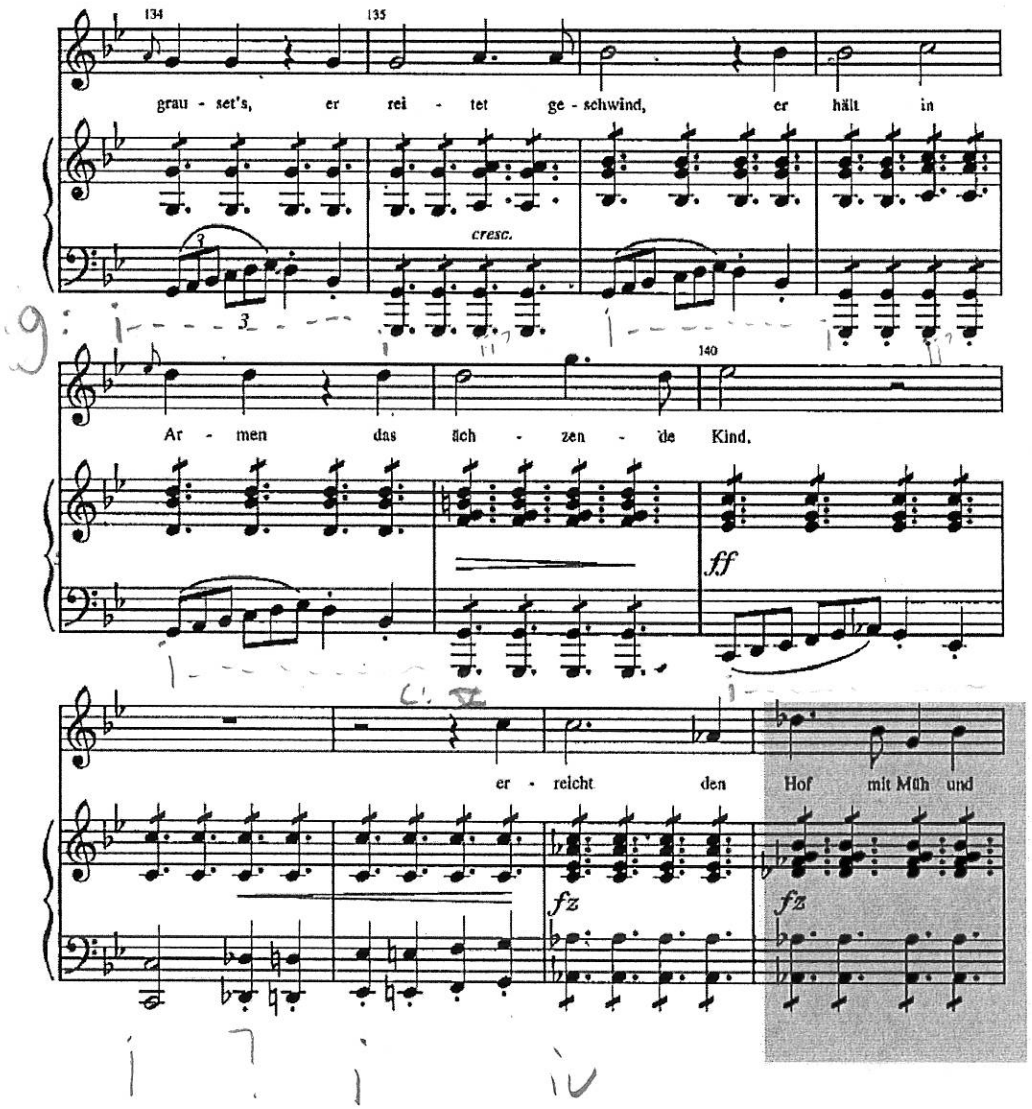
L1972

33a: Schubert "Erlkonig", mm. 113-23



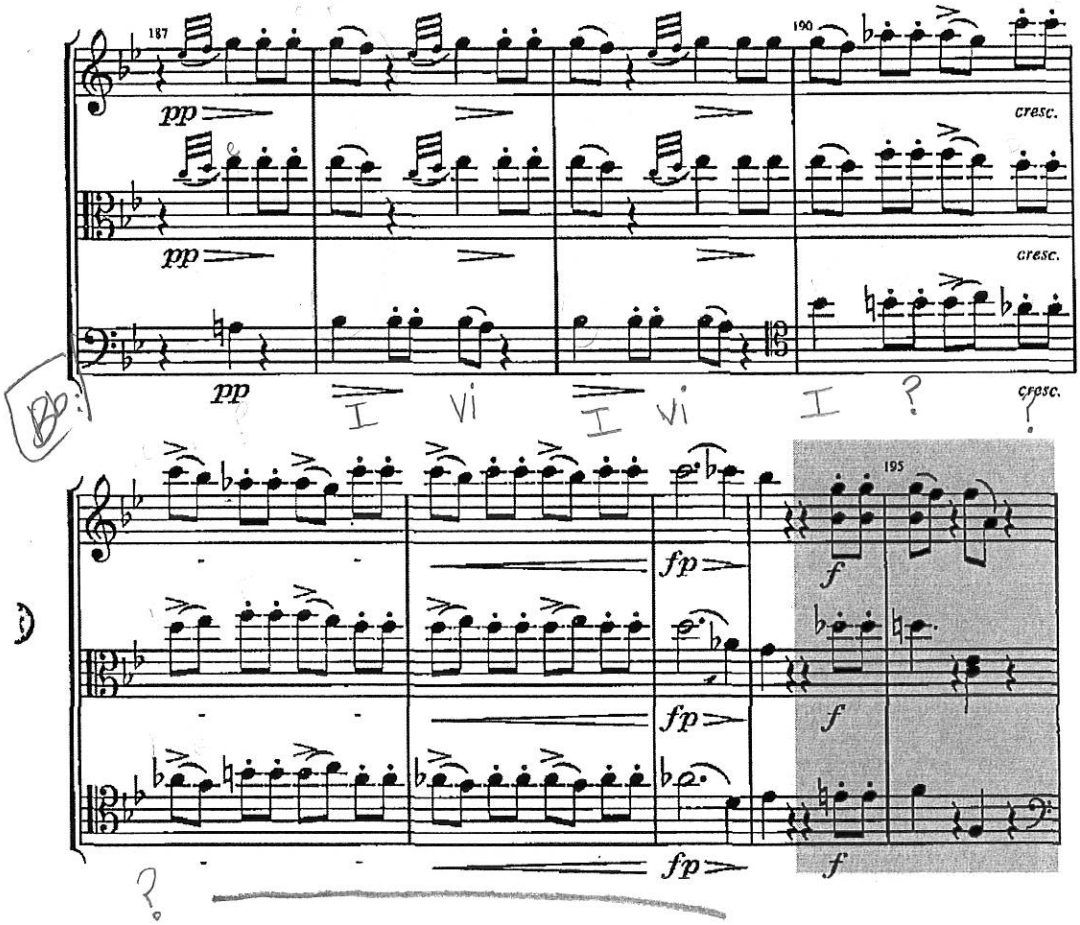
7035

34a: Schubert, "Erlkonig", mm. 134-48



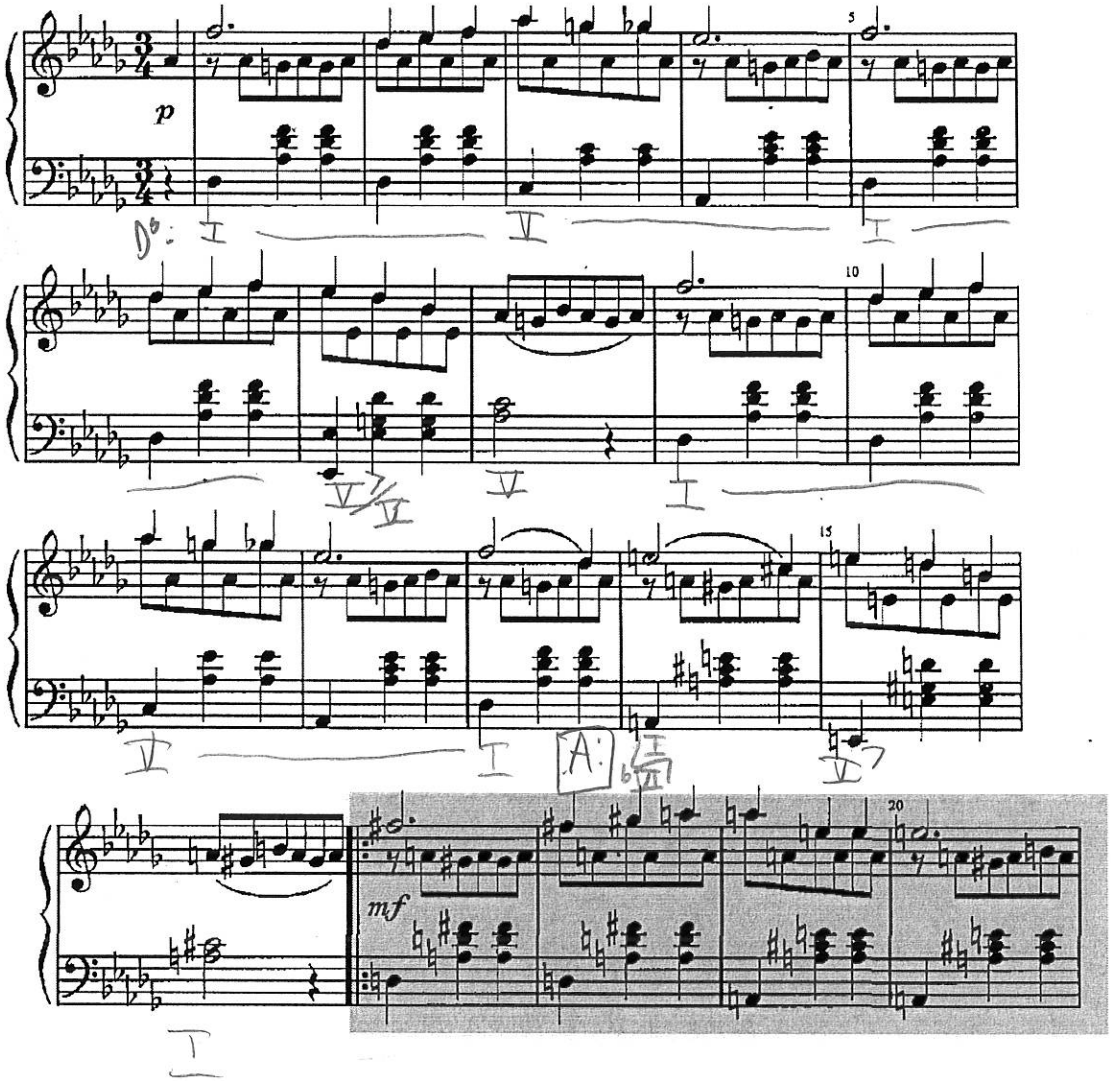
8516

37a: Schubert, String Trio D. 471



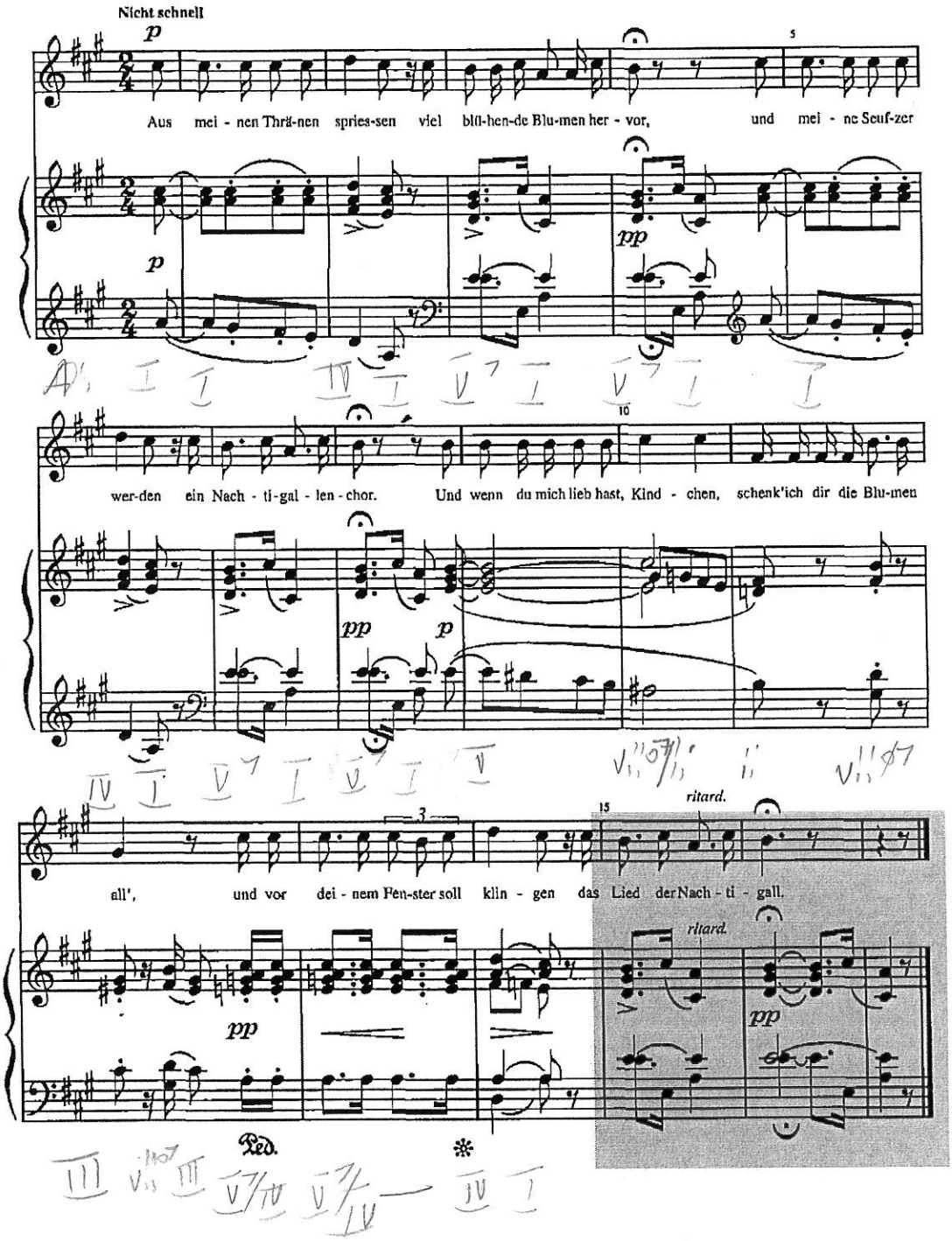
1231

38a: Schubert, Originaltaenze, op. 9, no. 14.



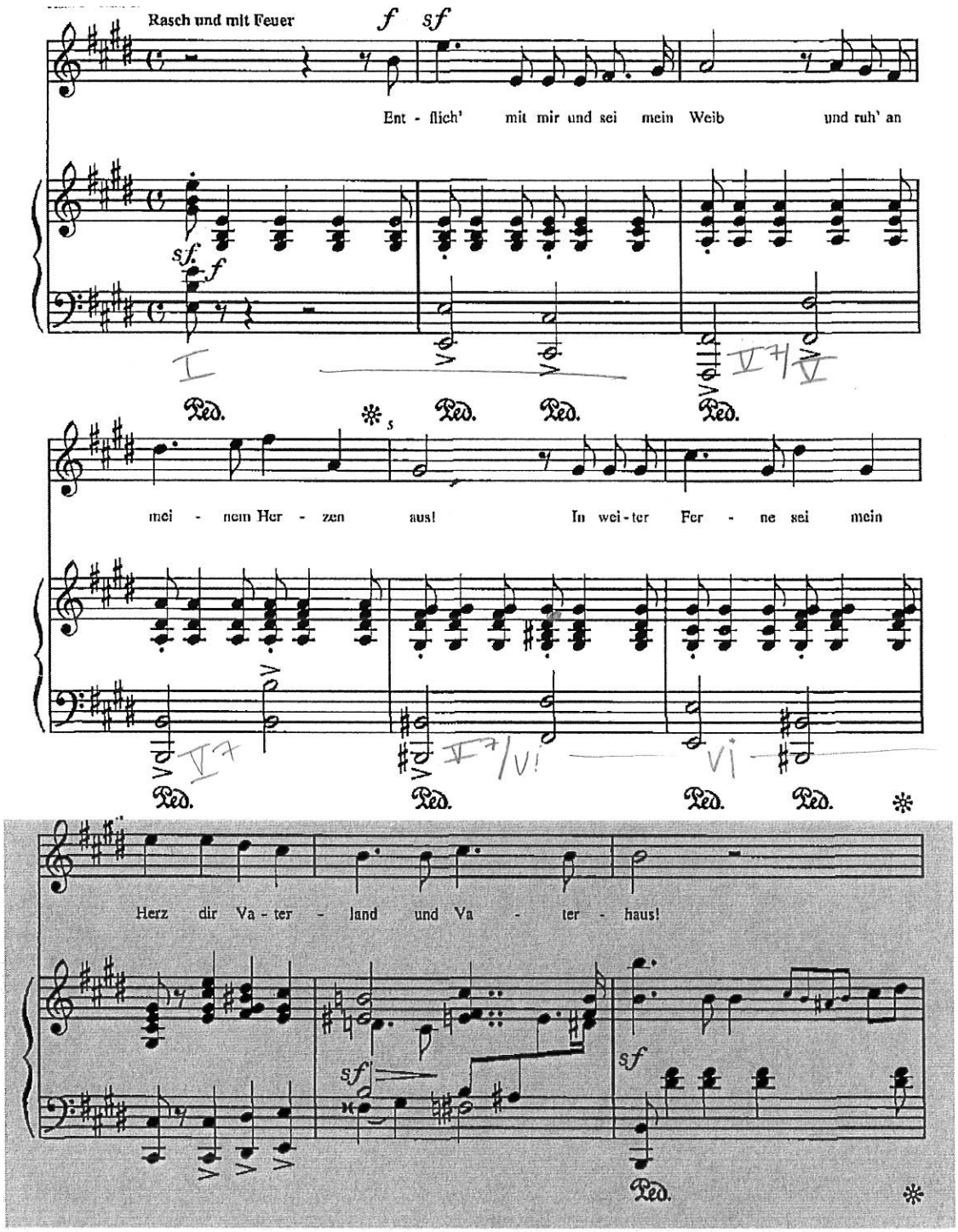
4762

41a: Schumann, "Aus meinen Thraenen spriessen," op. 48, no. 2



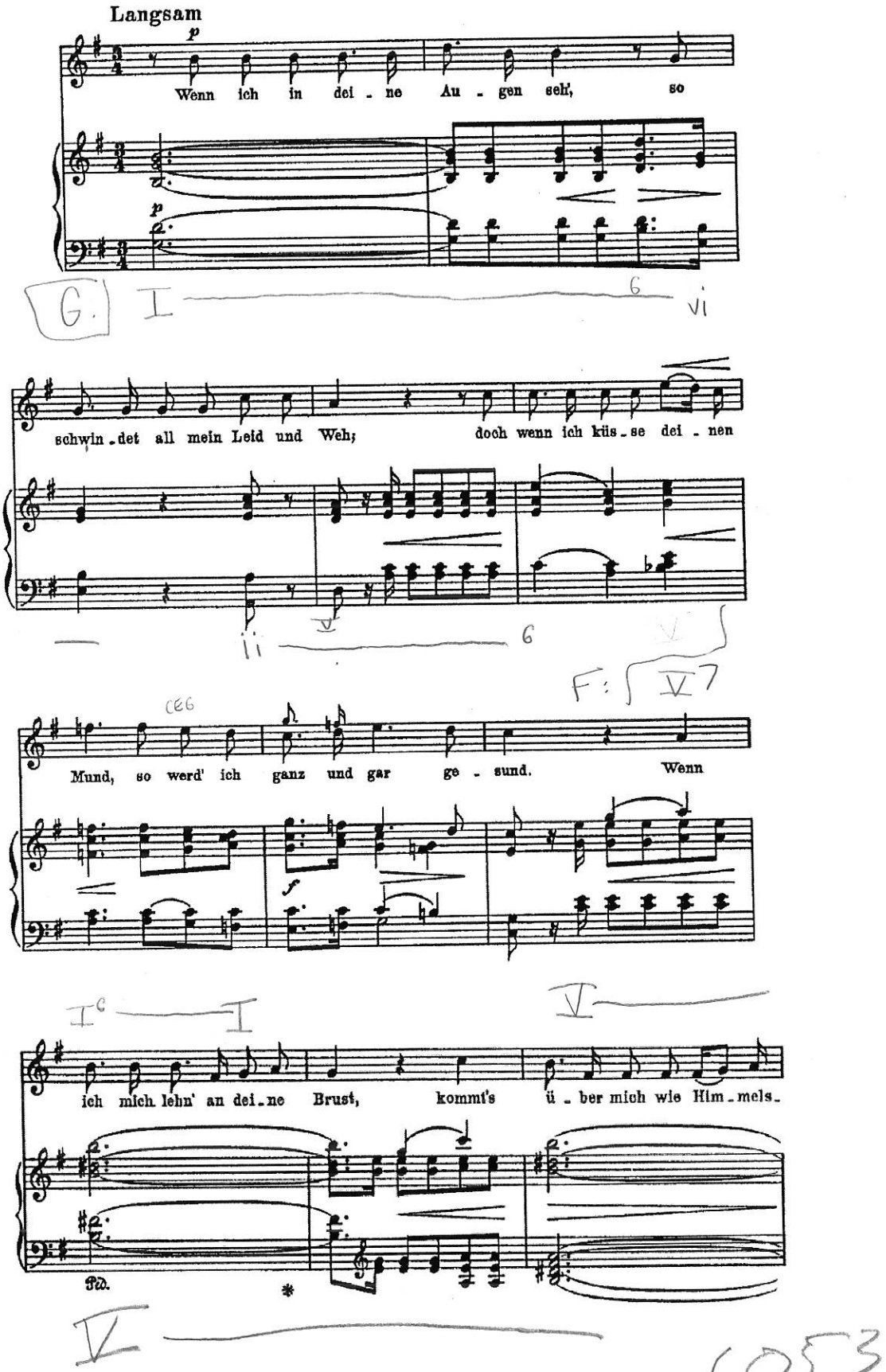
8530

42a: Schumann, Tragoedie, op. 136, no. 3

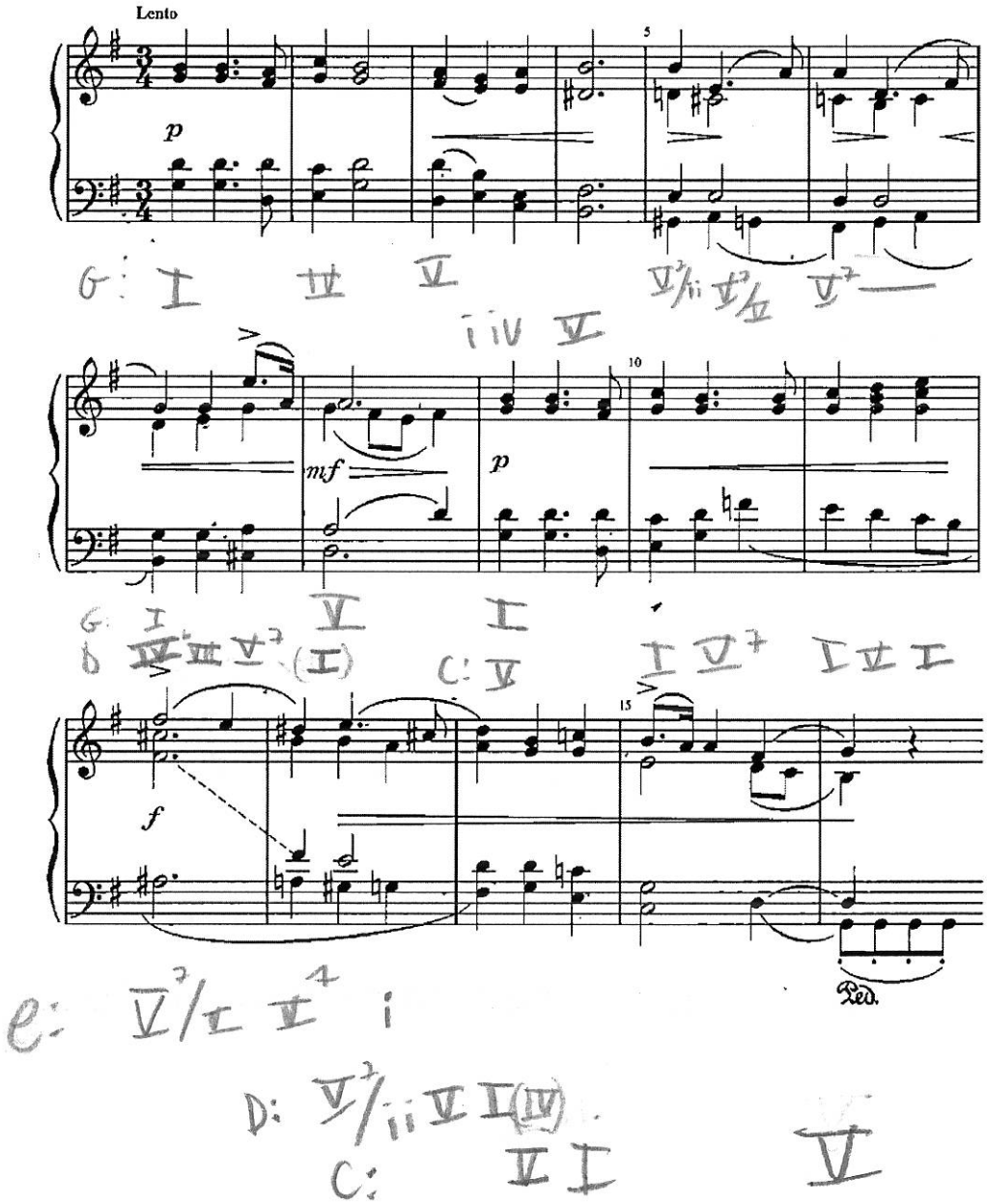


0656

43a: Schumann, "Wenn ich in deine Augen seh'"



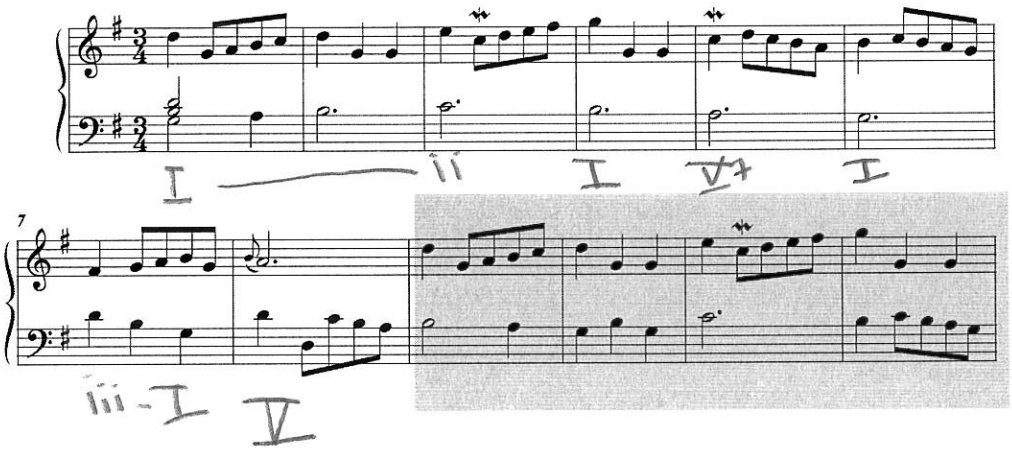
44a: Tchaikovsky, "Morning Prayer," op. 39, no. 1



7082

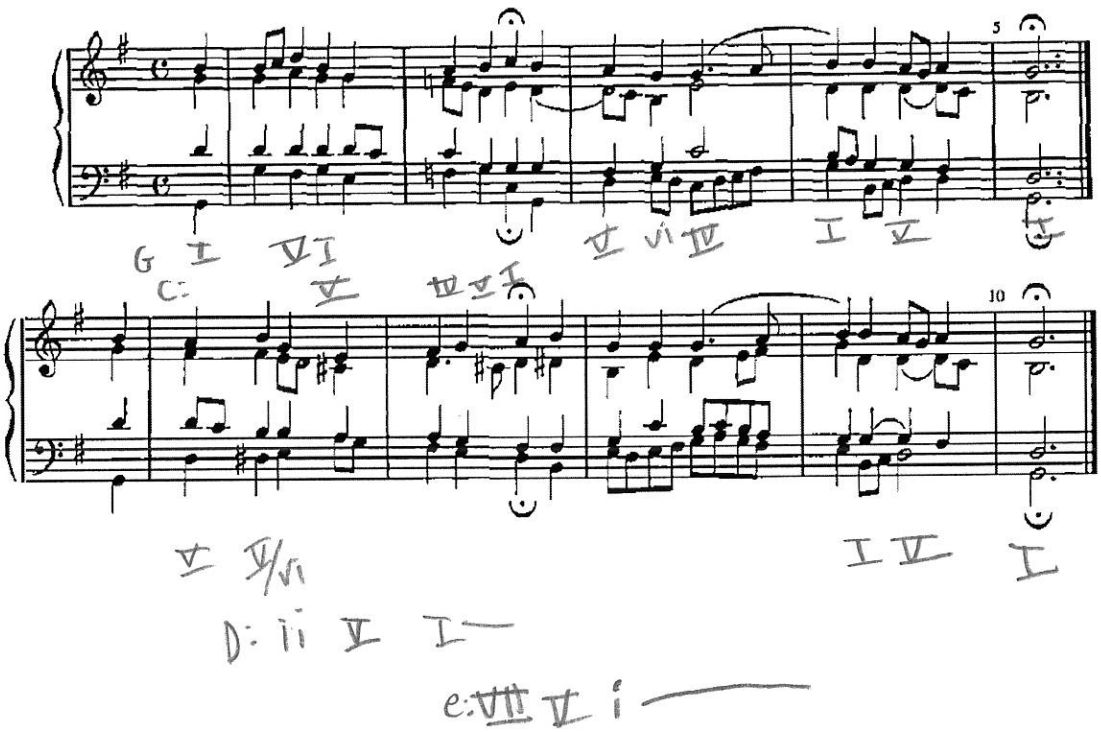
The following 32 analyses were created by the reduced YCAC model, with parameters set as described in the above study.

Ex 1a, Anonymous (but often attributed to Bach), Minuet in G



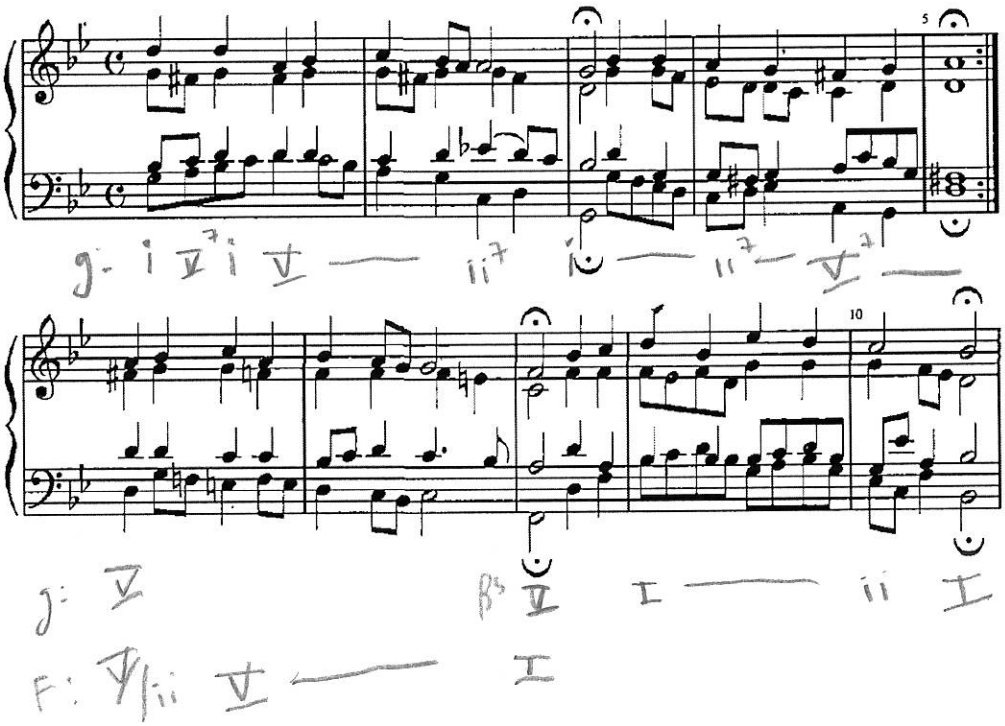
8010

2a Bach, Chorale, "Uns ist ein Kindlein heut' geboren"



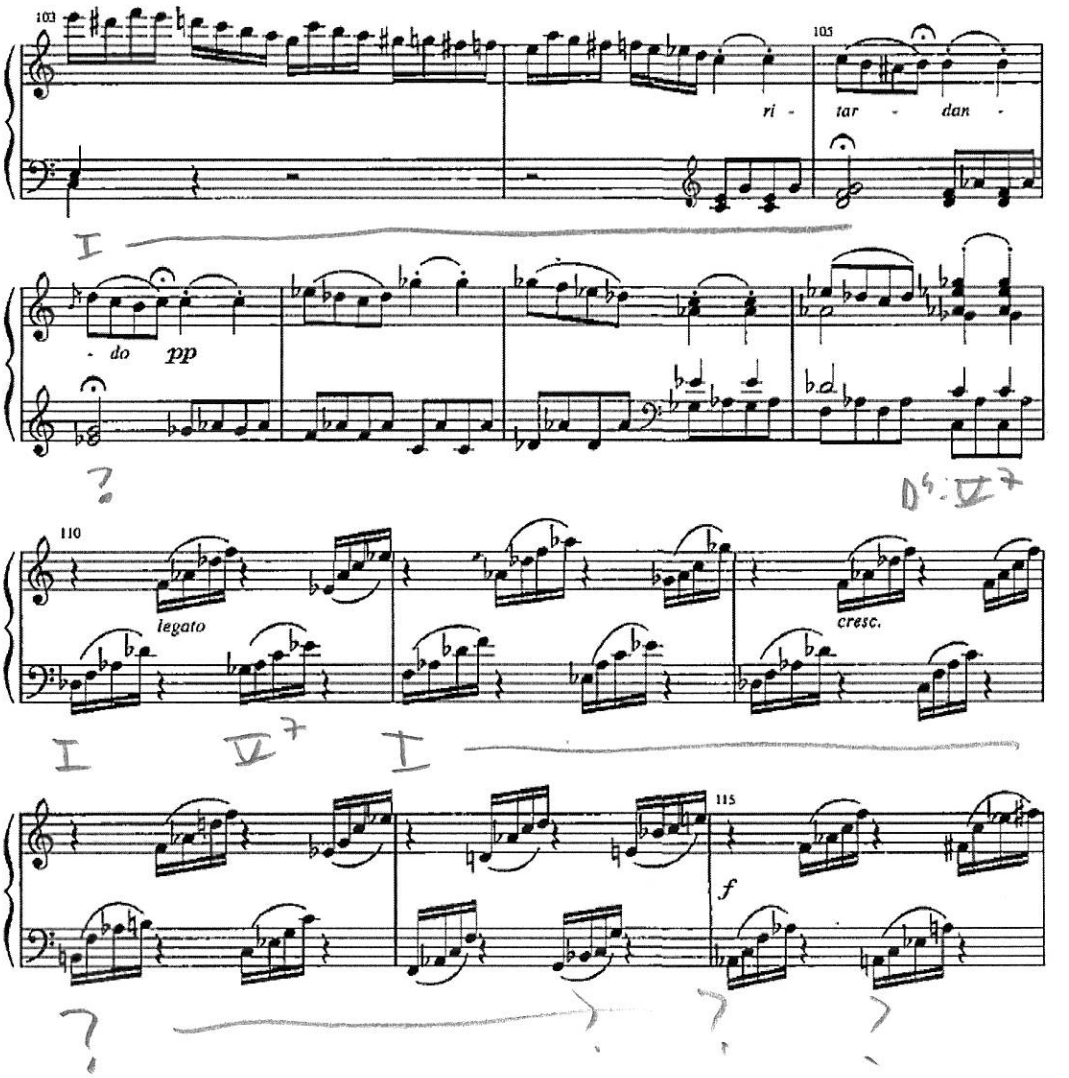
8214

3a: Bach, “Jesu, der du meine Seele”



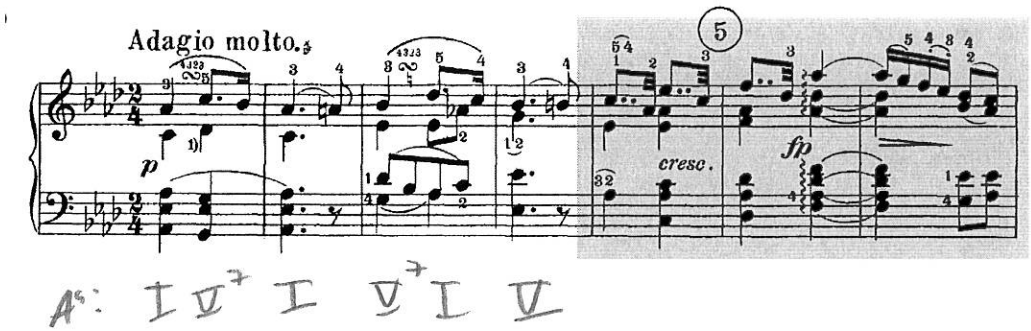
4685

4a: Beethoven, Rondo op. 51, no. 1



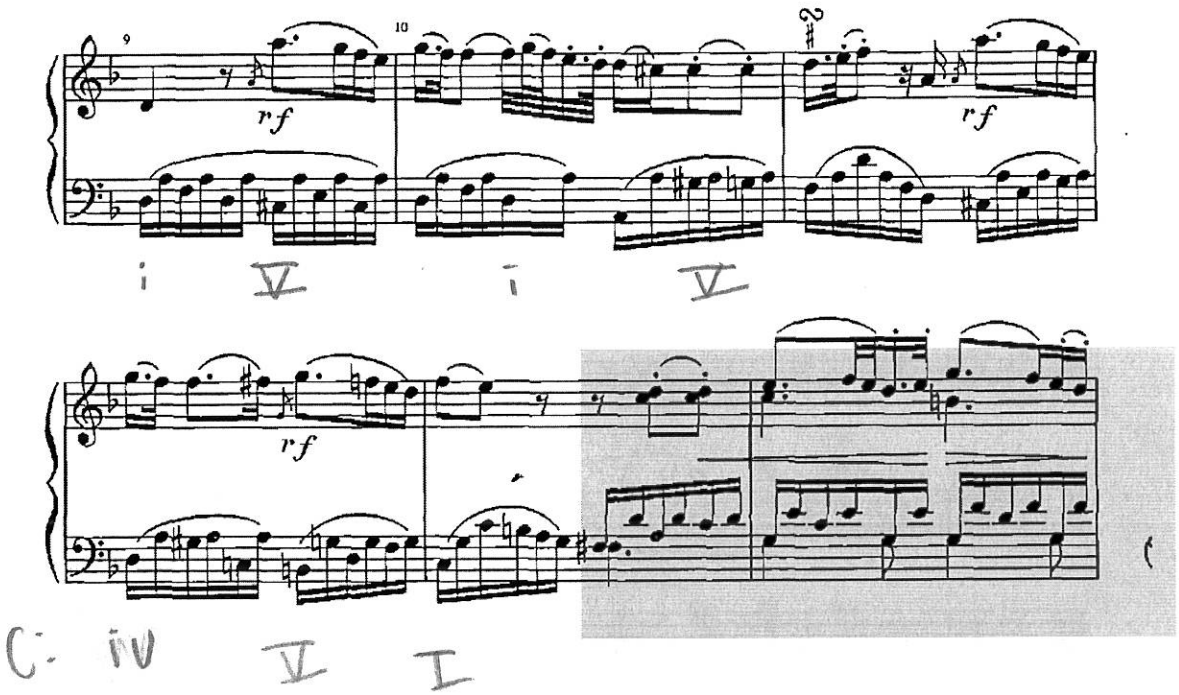
9744

5a: Beethoven, Sonata Op. 10 No. 1, ii



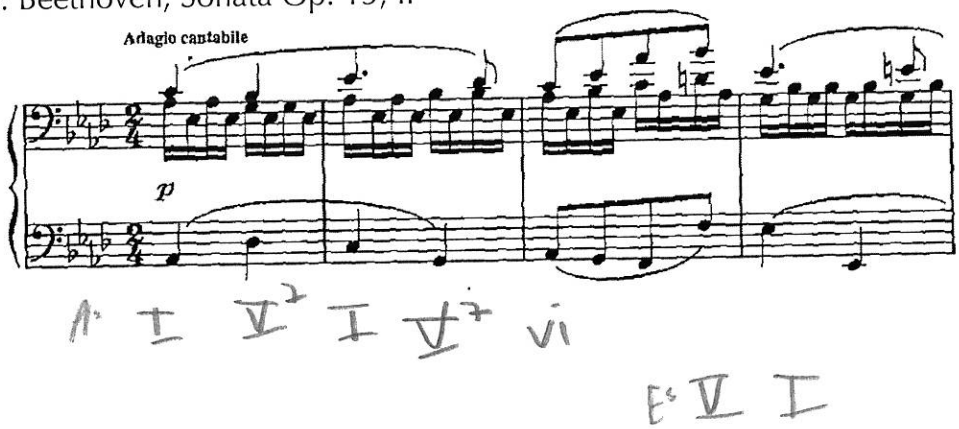
6482

6a: Beethoven, Piano Sonata op. 10, no. 3, ii



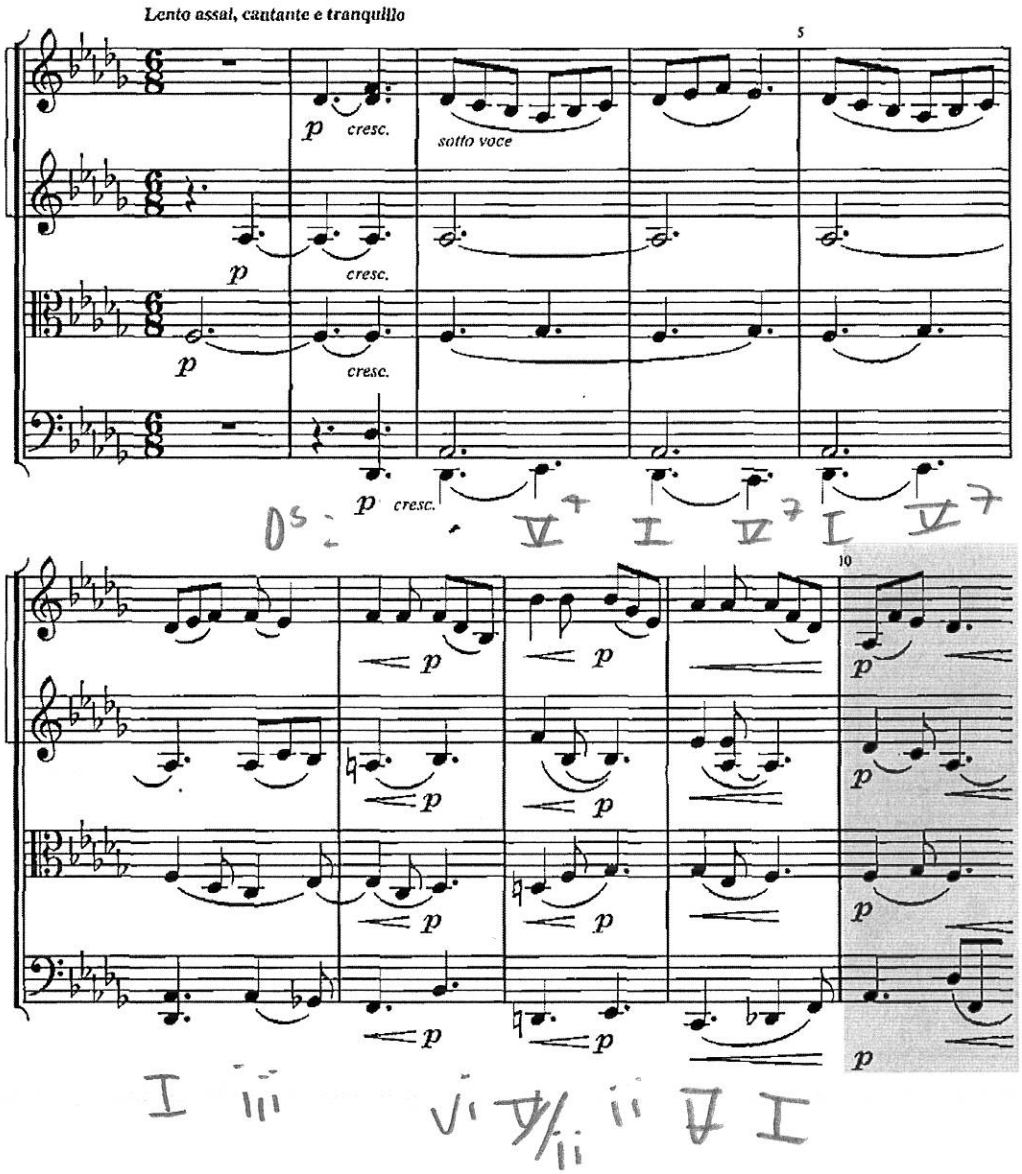
9855

7a: Beethoven, Sonata Op. 13, II



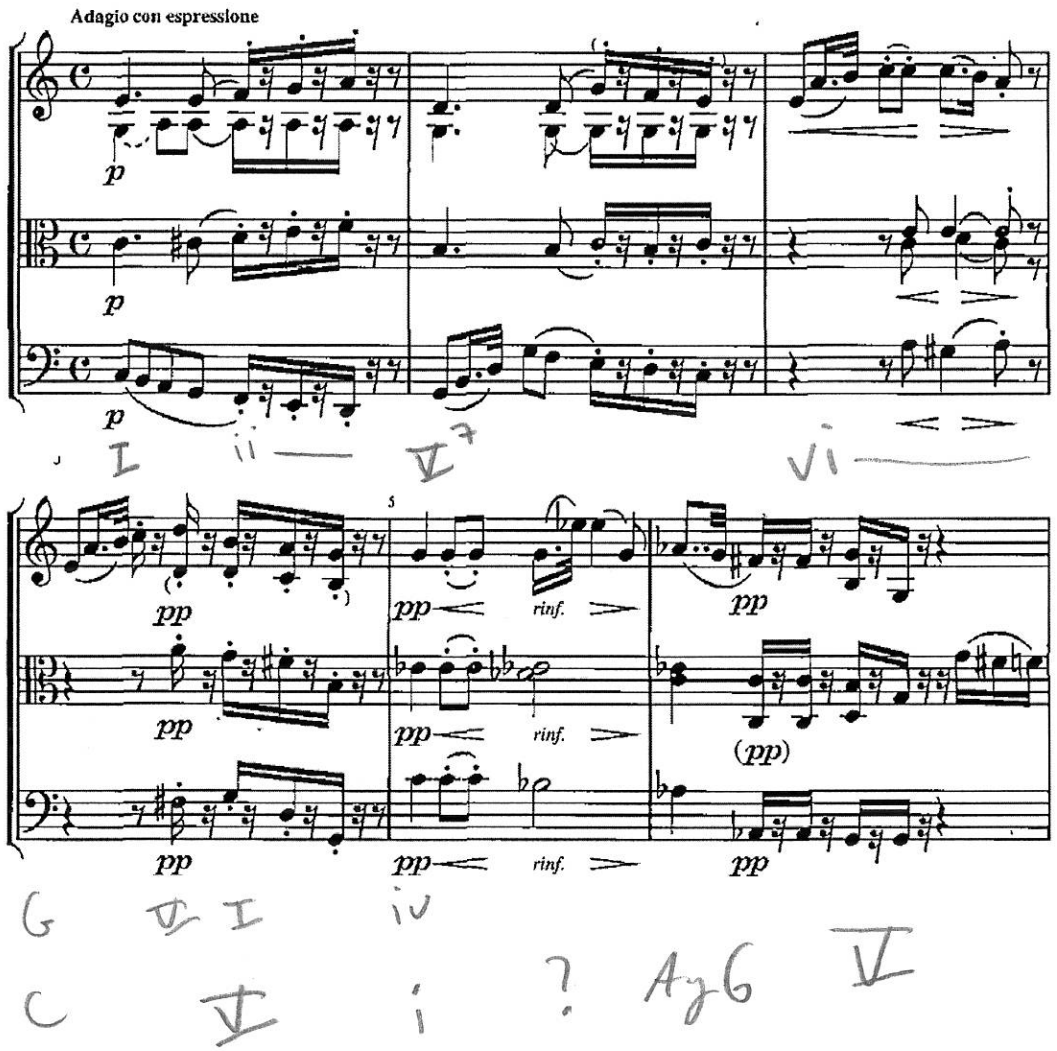
1025

11a: Beethoven, String Quartet op. 135, iii



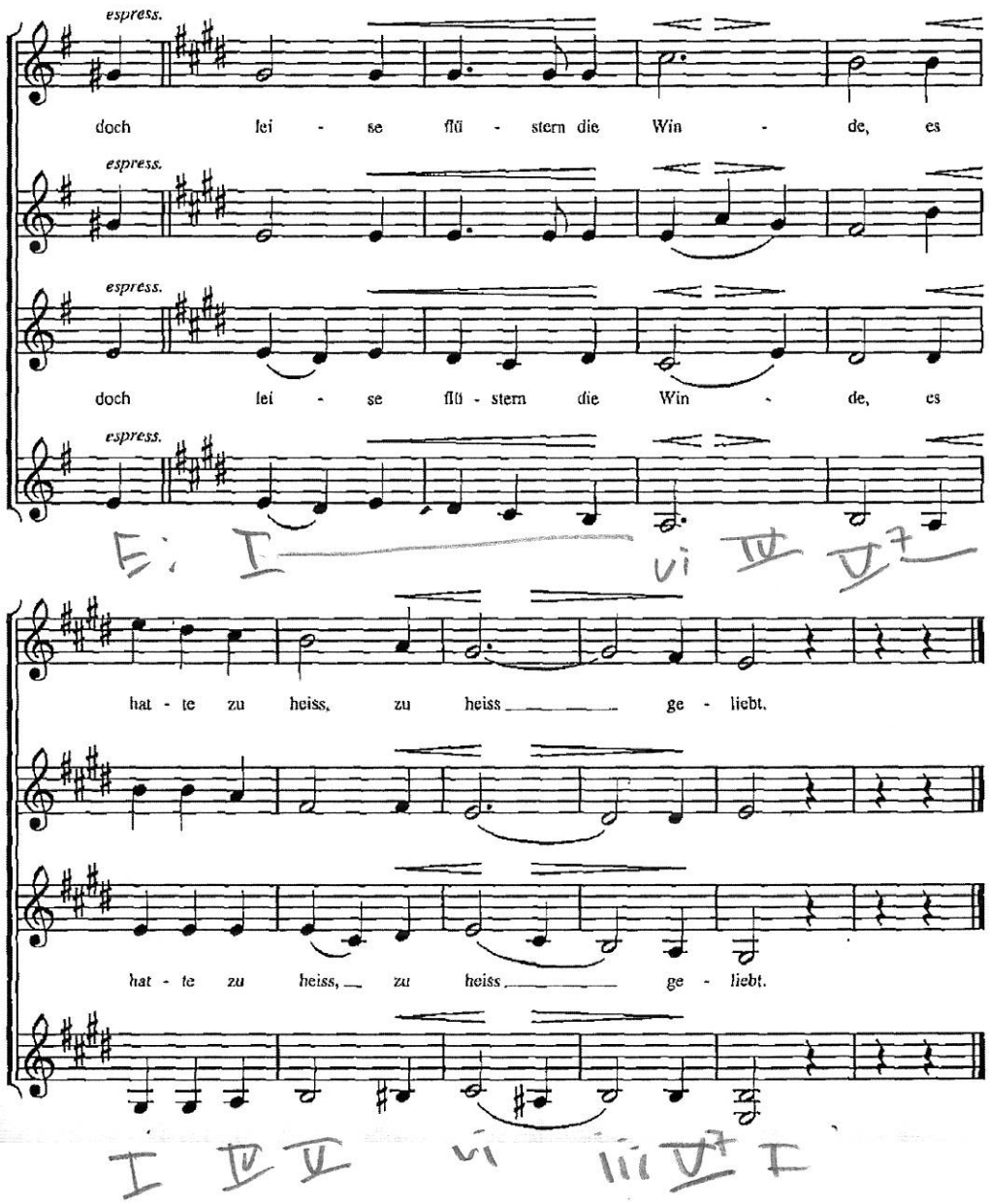
6898

12a: Beethoven String Trio op. 9, no. 3, ii



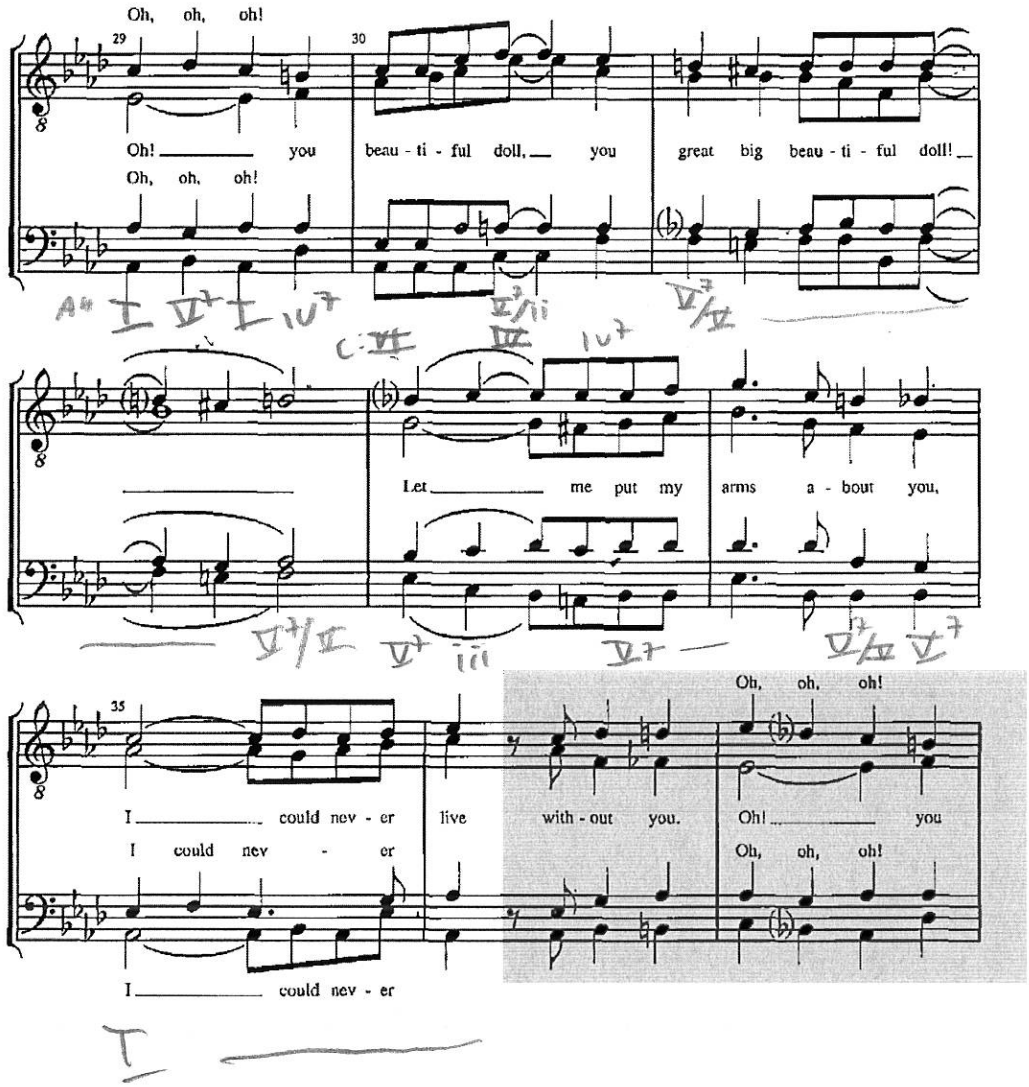
4316

13a: Brahms, "Und gehst du ueber den Kirchhof", op. 44



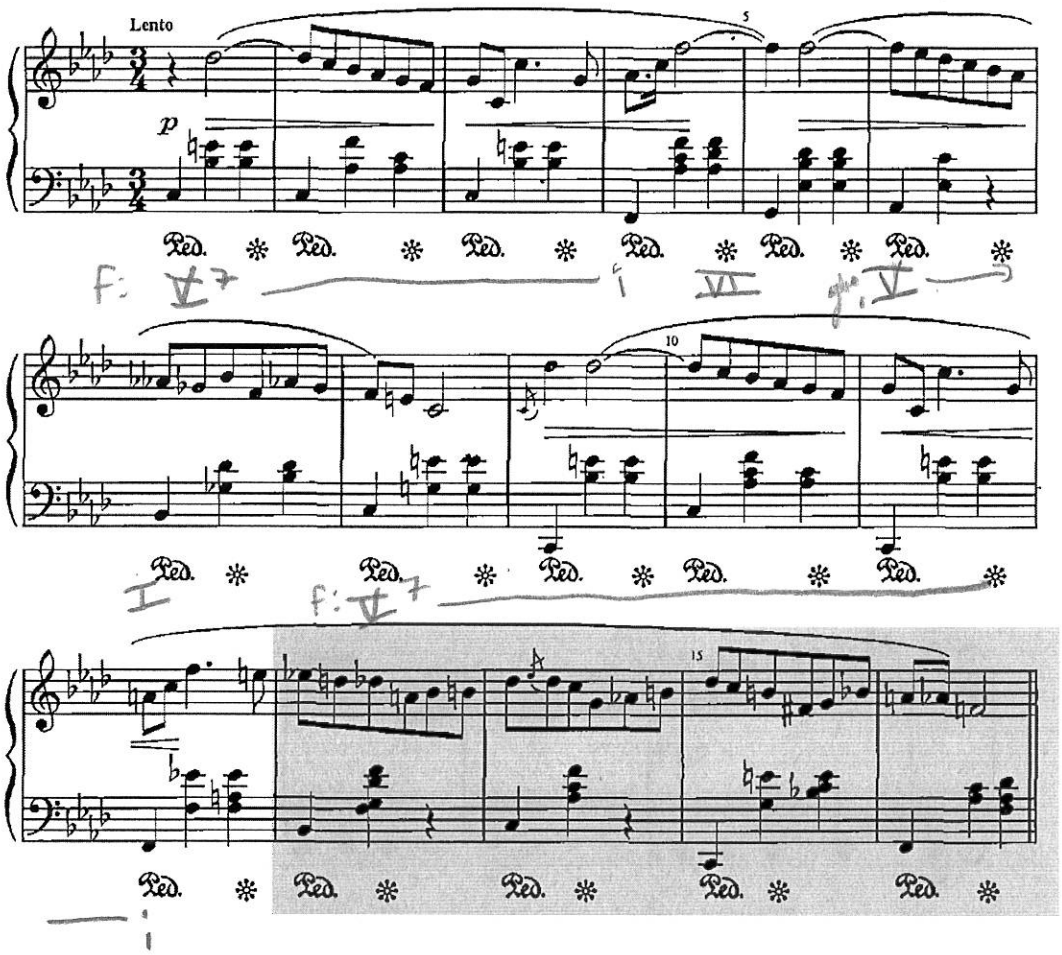
5050

14a: Ayer (arr. by Campbell), "Oh! You Beautiful Doll"



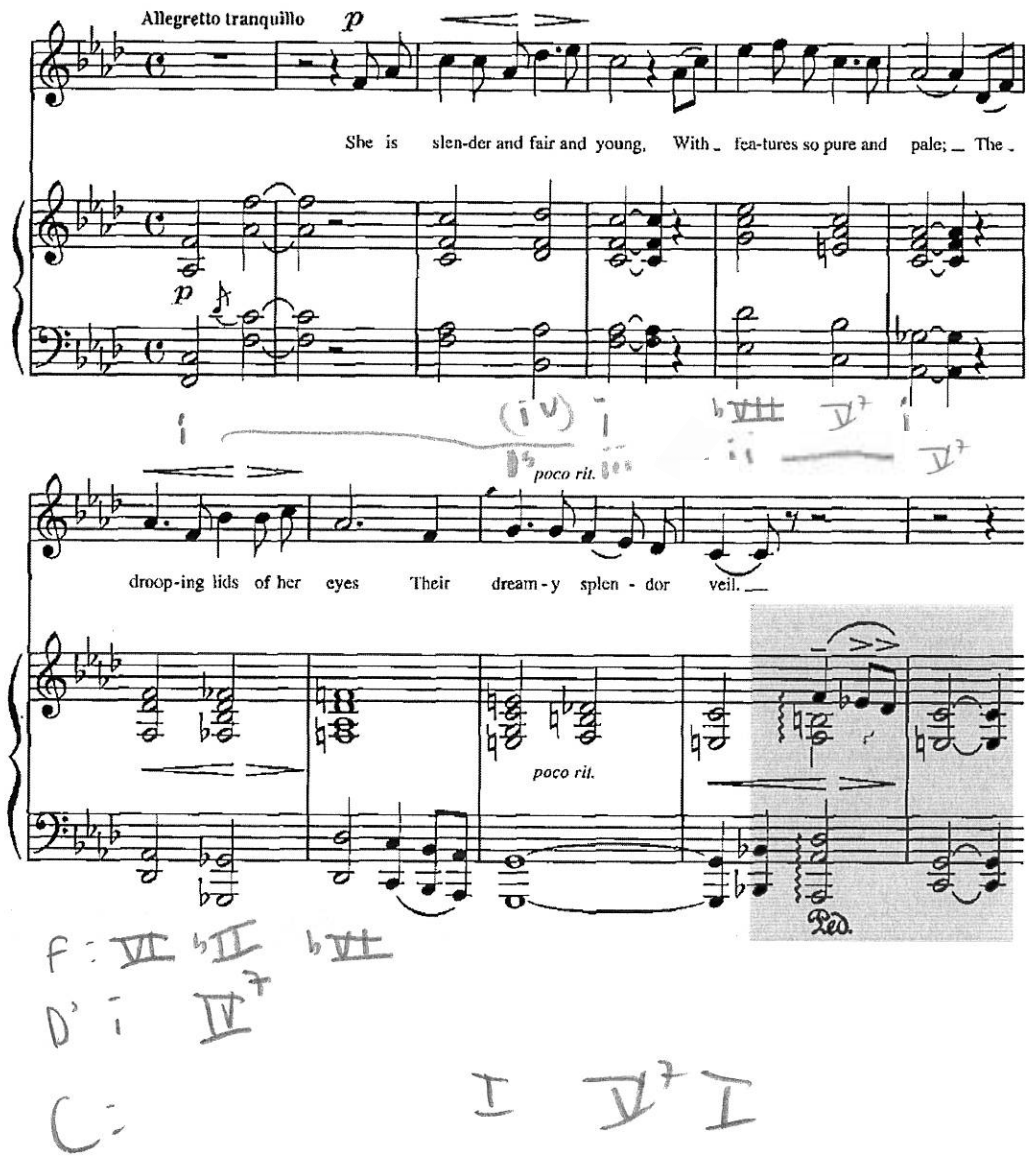
6465

16a: Chopin, Mazurka op. 63, no. 2



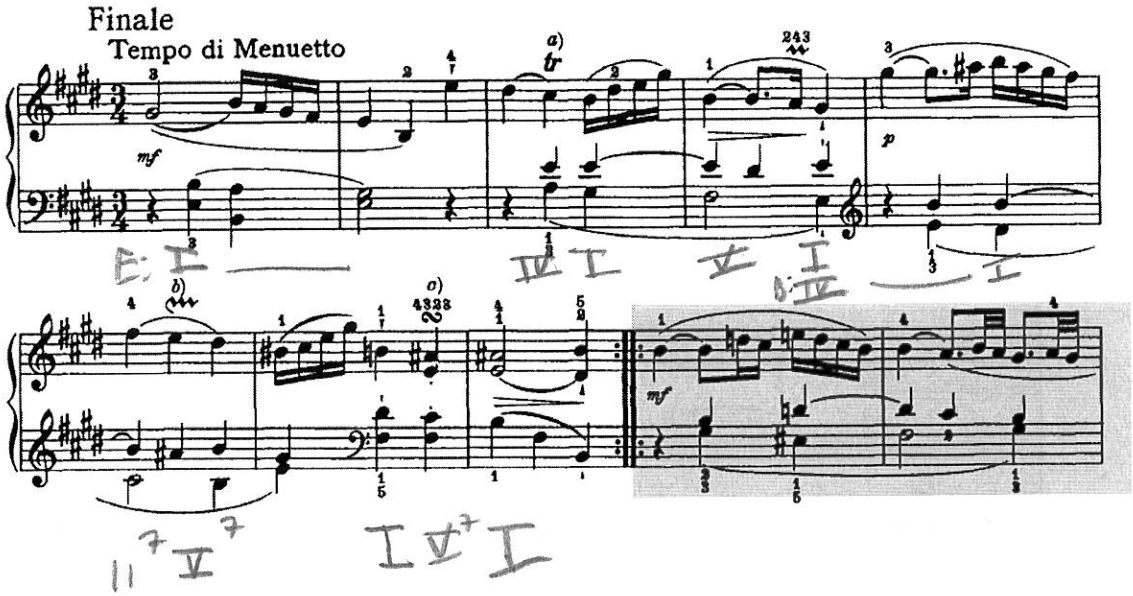
5068

18a: Grieg, "The Mountain Maid," op. 67, no. 2



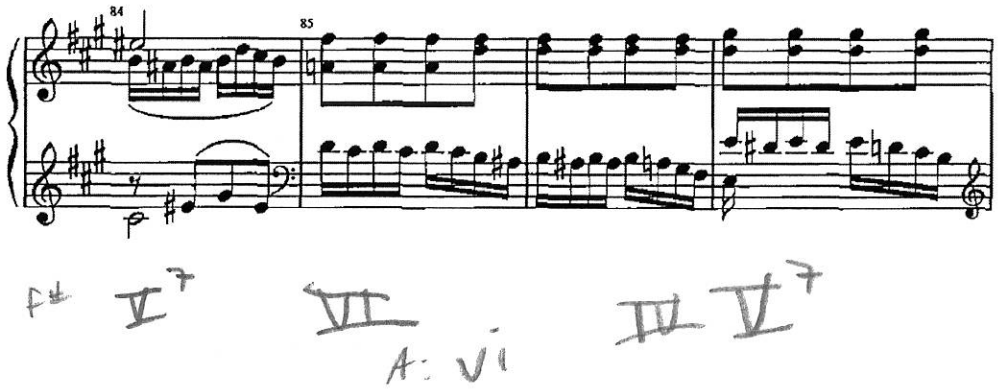
2982

Ex19a: Haydn, Sonata No. 22, iii



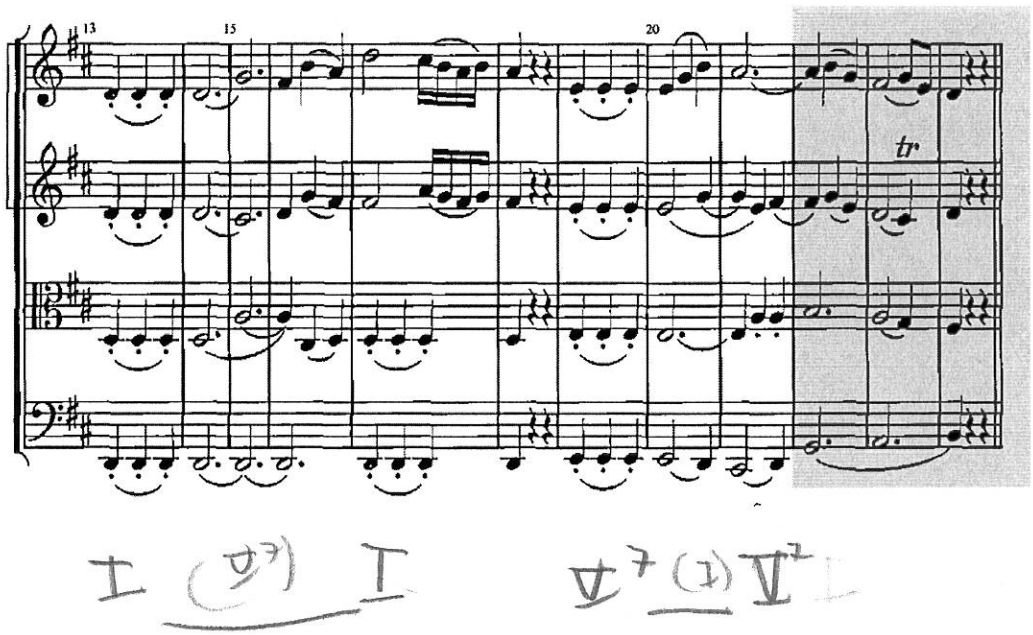
5577

20a: Haydn, Piano Sonata no 30, I



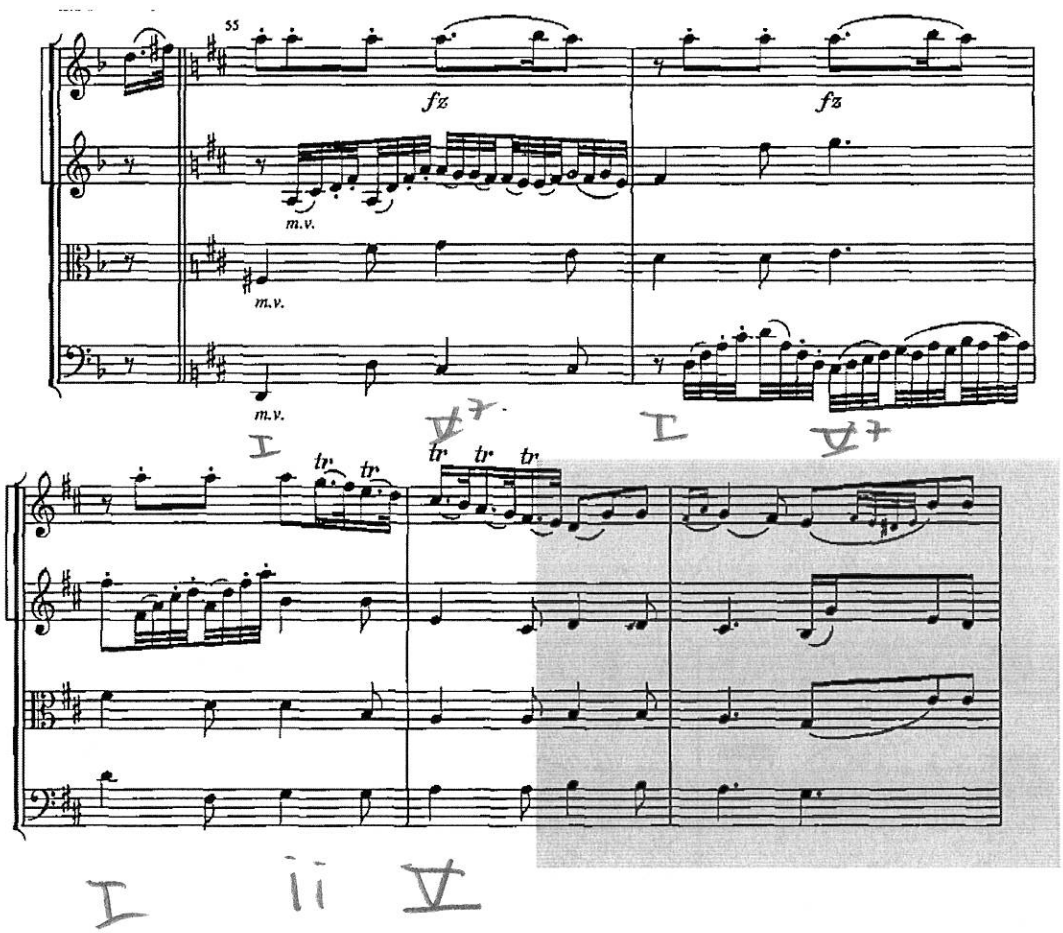
3076

21a: Haydn, String Quartet op. 20, no. 4, i



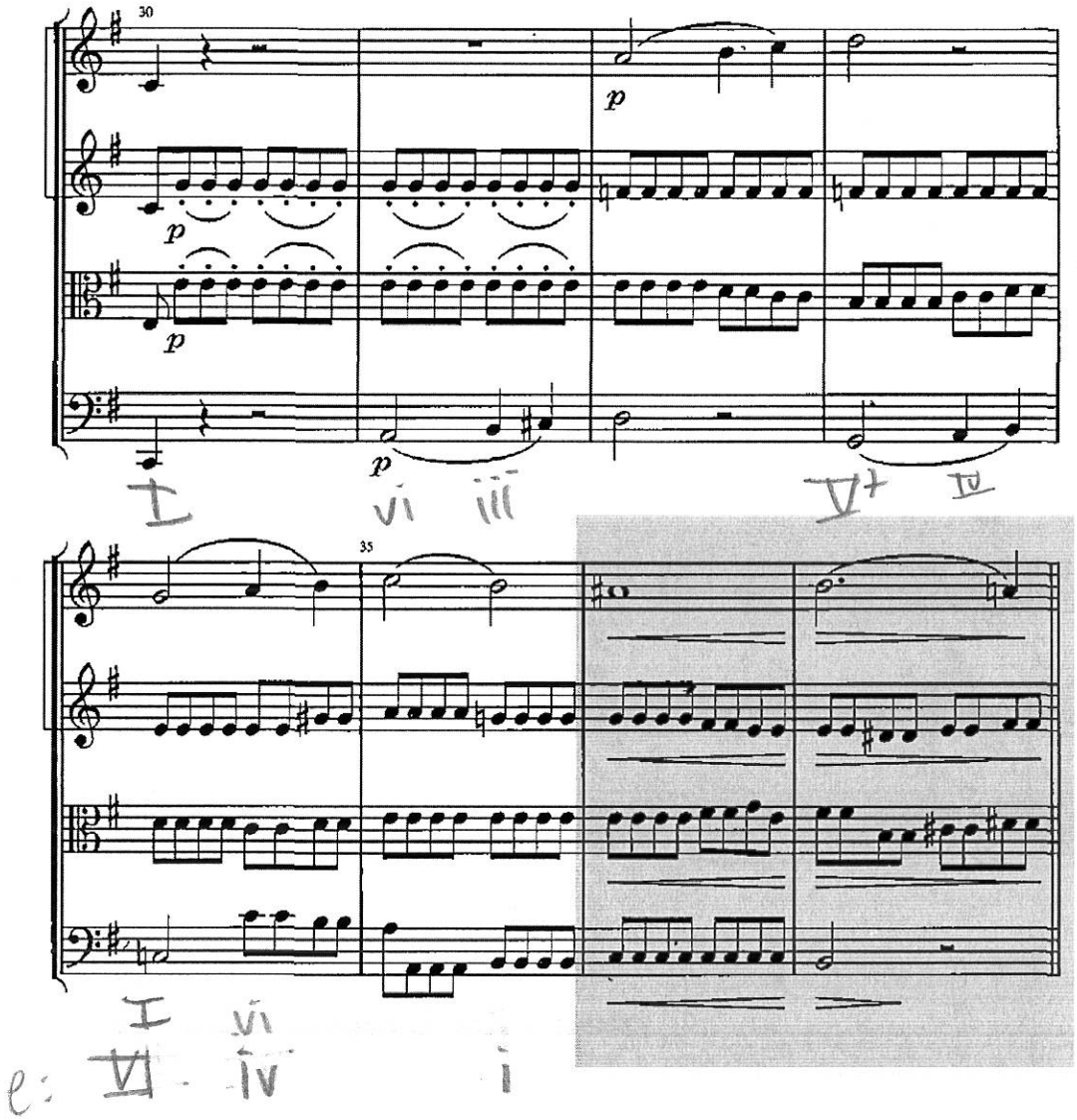
4416

22a: Haydn, String Quartet op 50, no 6 ii



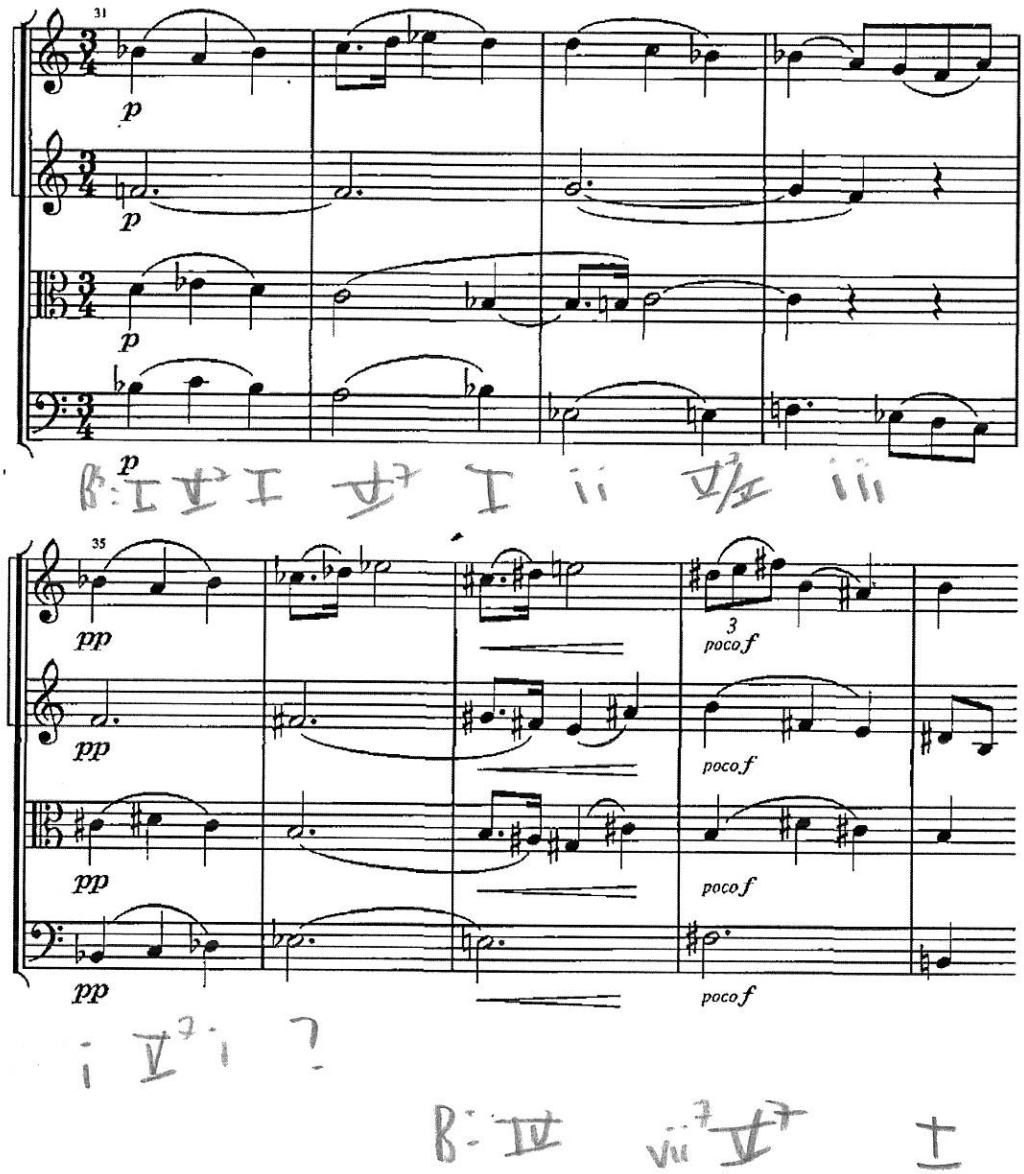
2628

23a: Haydn, String Quartet op. 74, no 3, ii



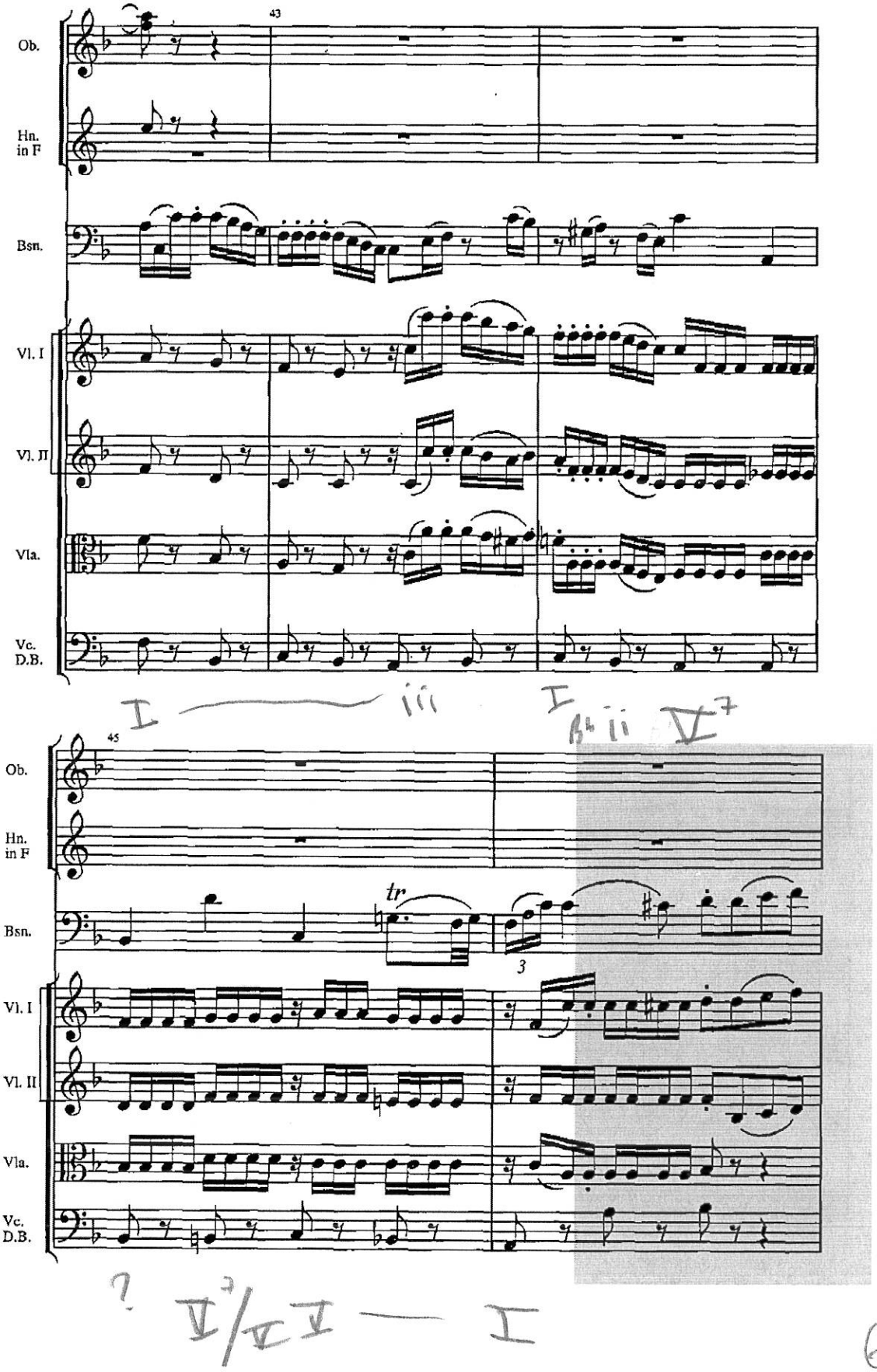
9405

24a: Haydn, String Quartet op. 76, no. 6, ii



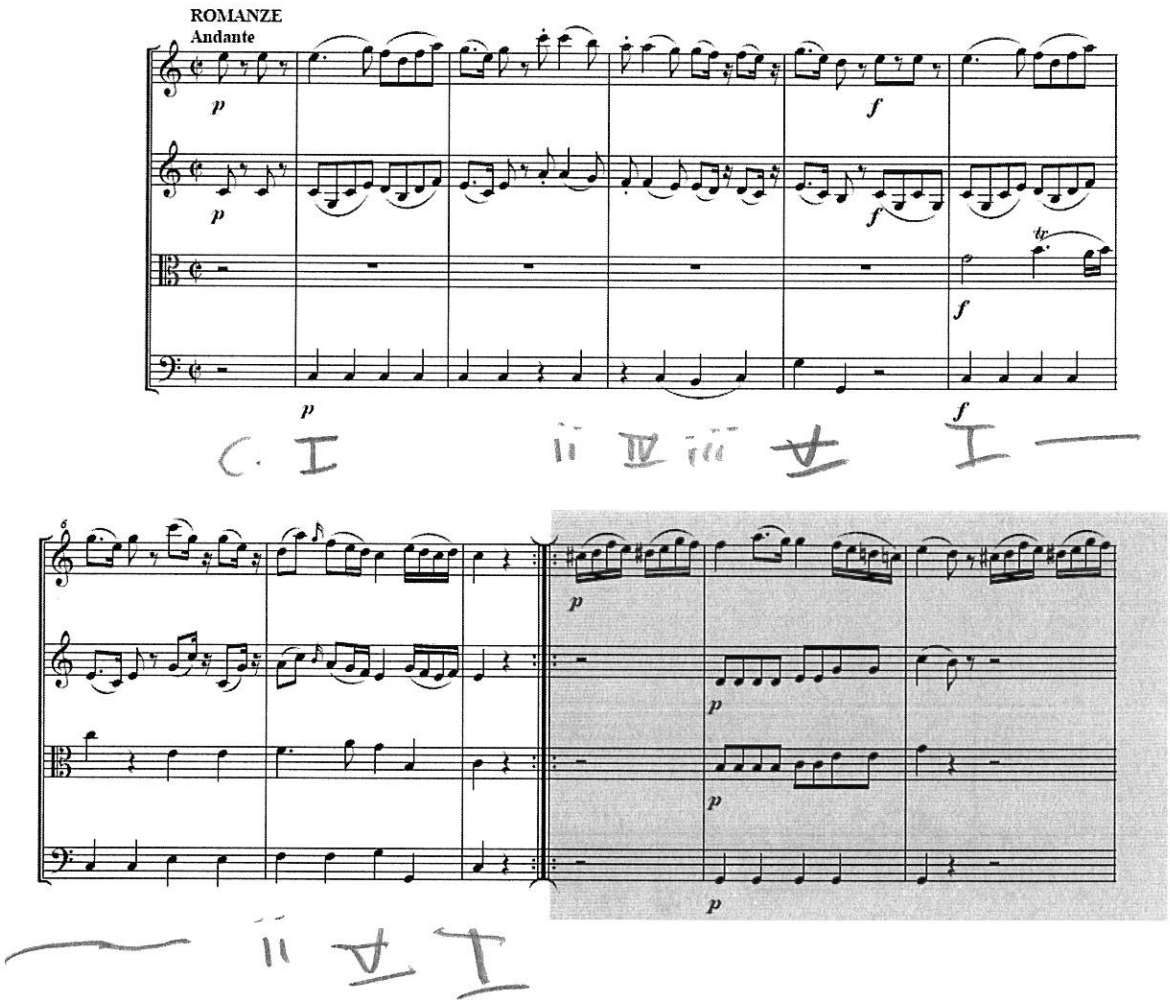
7167

25a: Mozart, Bassoon Concerto K. 191, ii



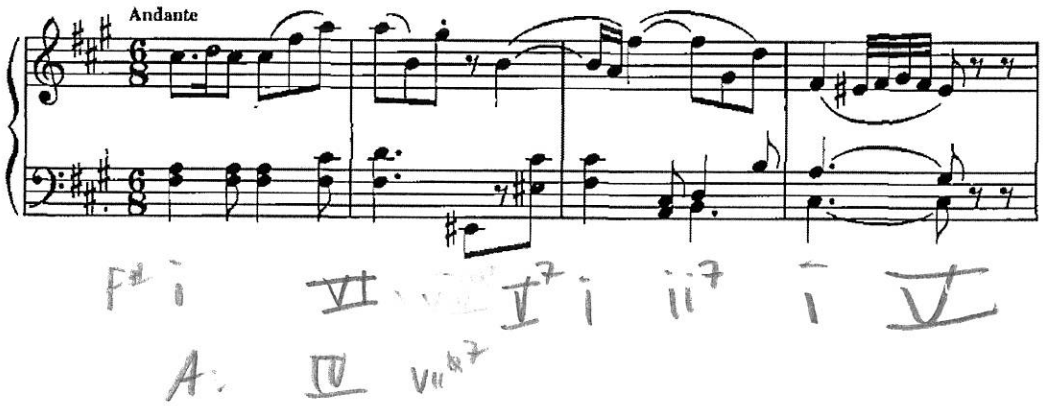
6085

Ex 26a. Mozart, "Eine Kleine Nachtmusik", K. 525, ii



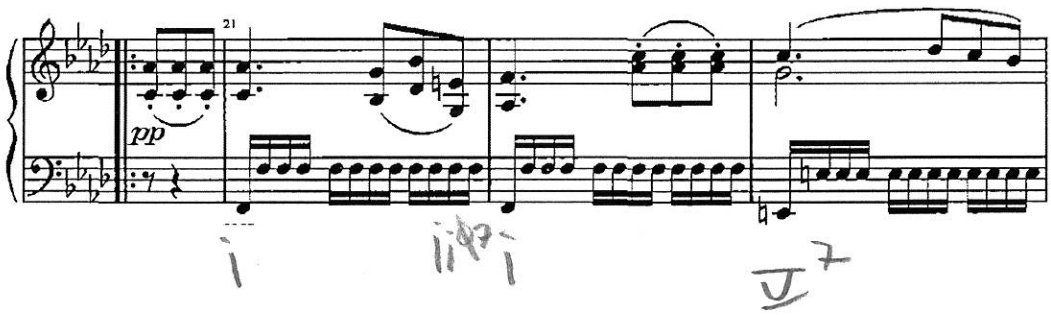
7641

27a: Mozart, Piano Concerto K. 488, ii



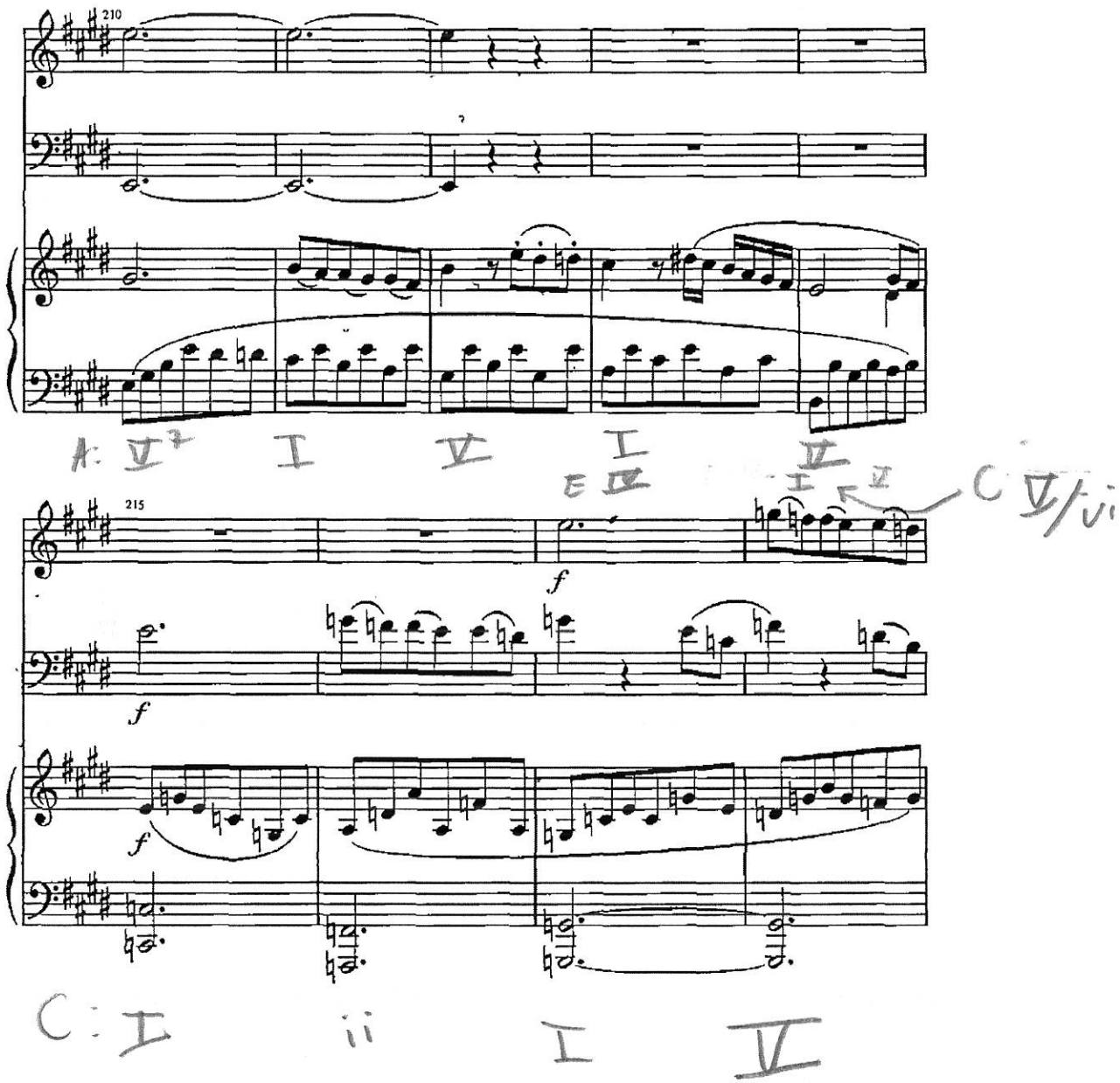
6109

28a: Mozart, Piano Sonata K. 330, ii



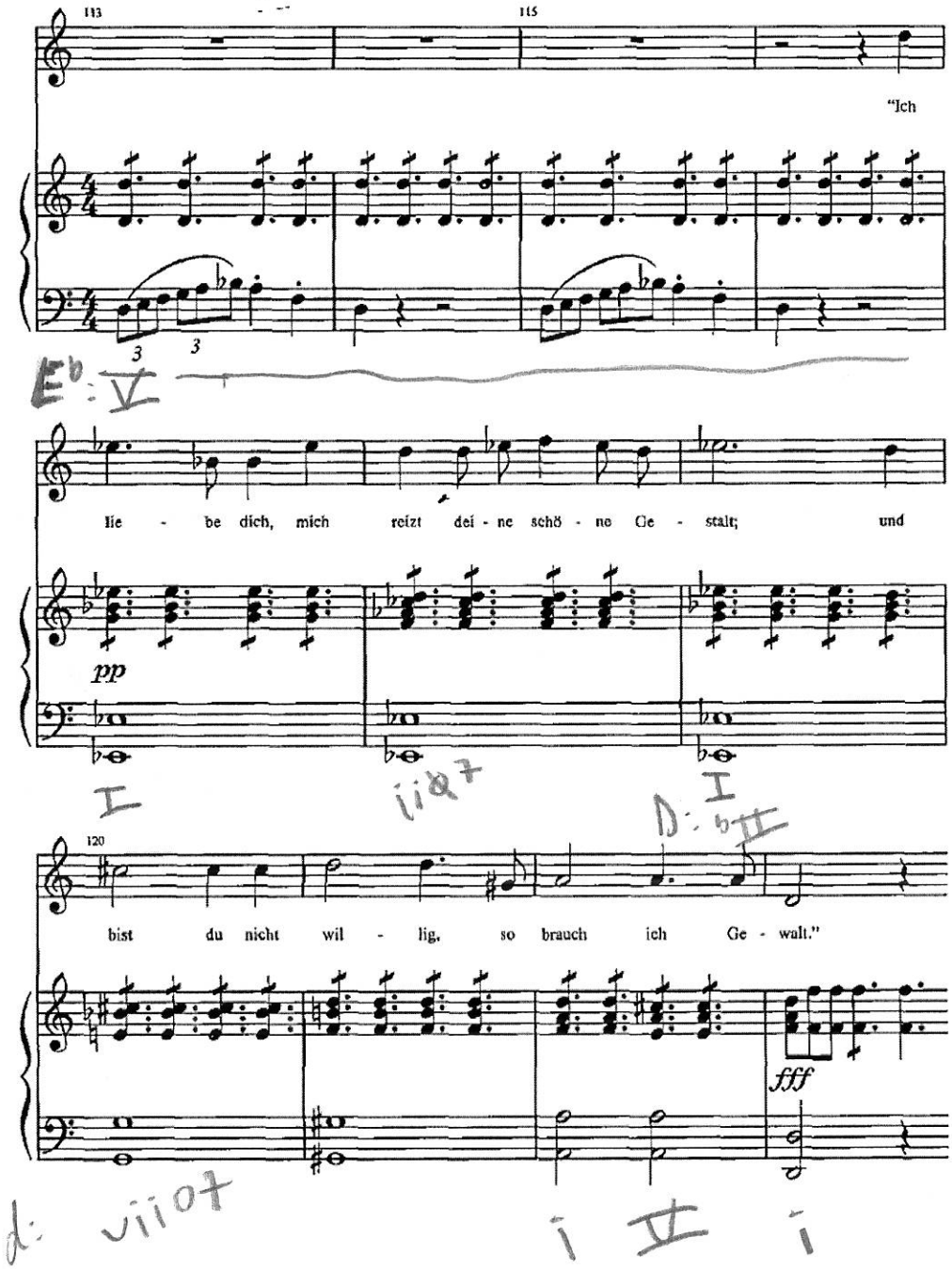
6980

30a: Mozart, Piano Trio K. 542, i



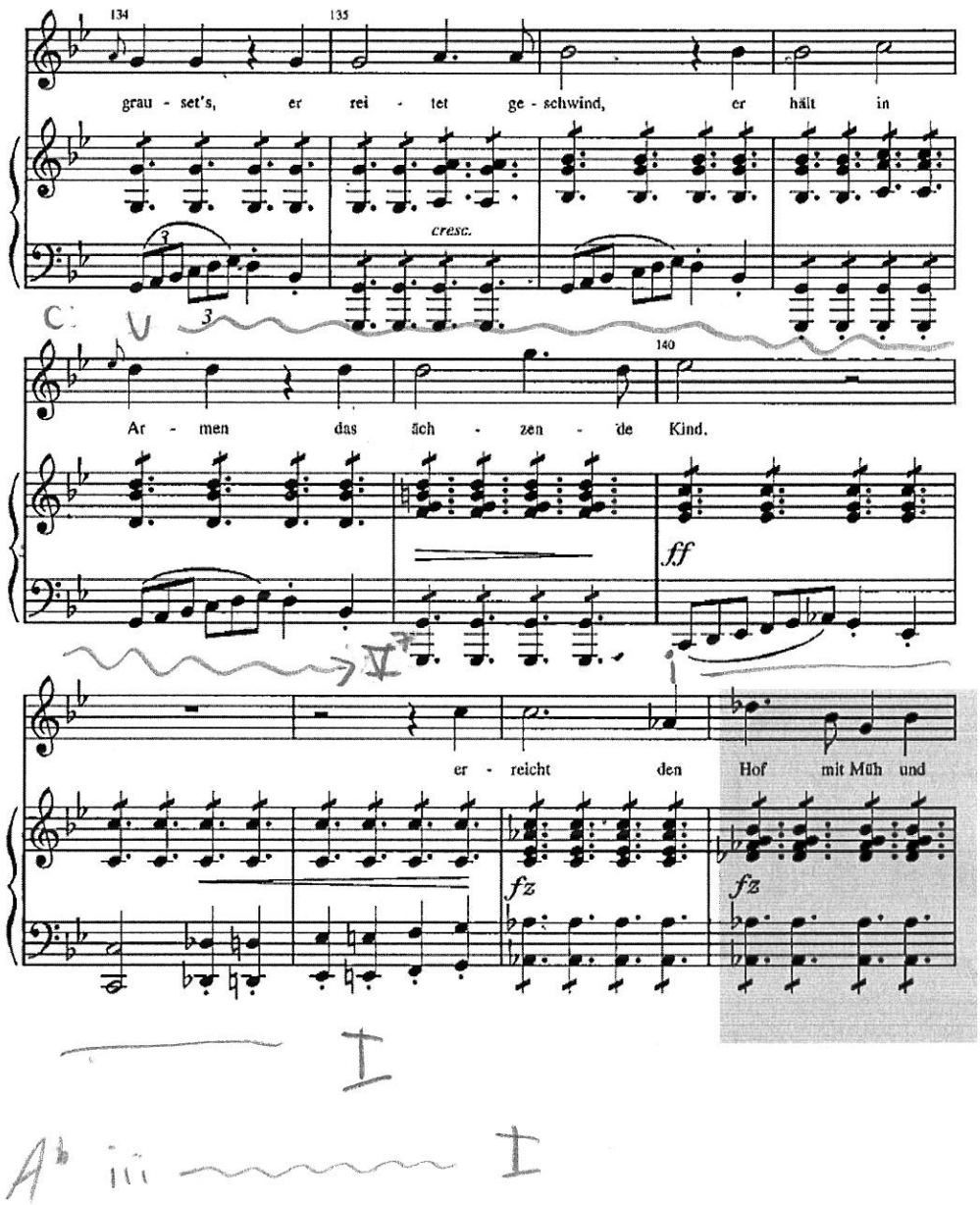
7915

33a: Schubert "Erlkonig", mm. 113-23



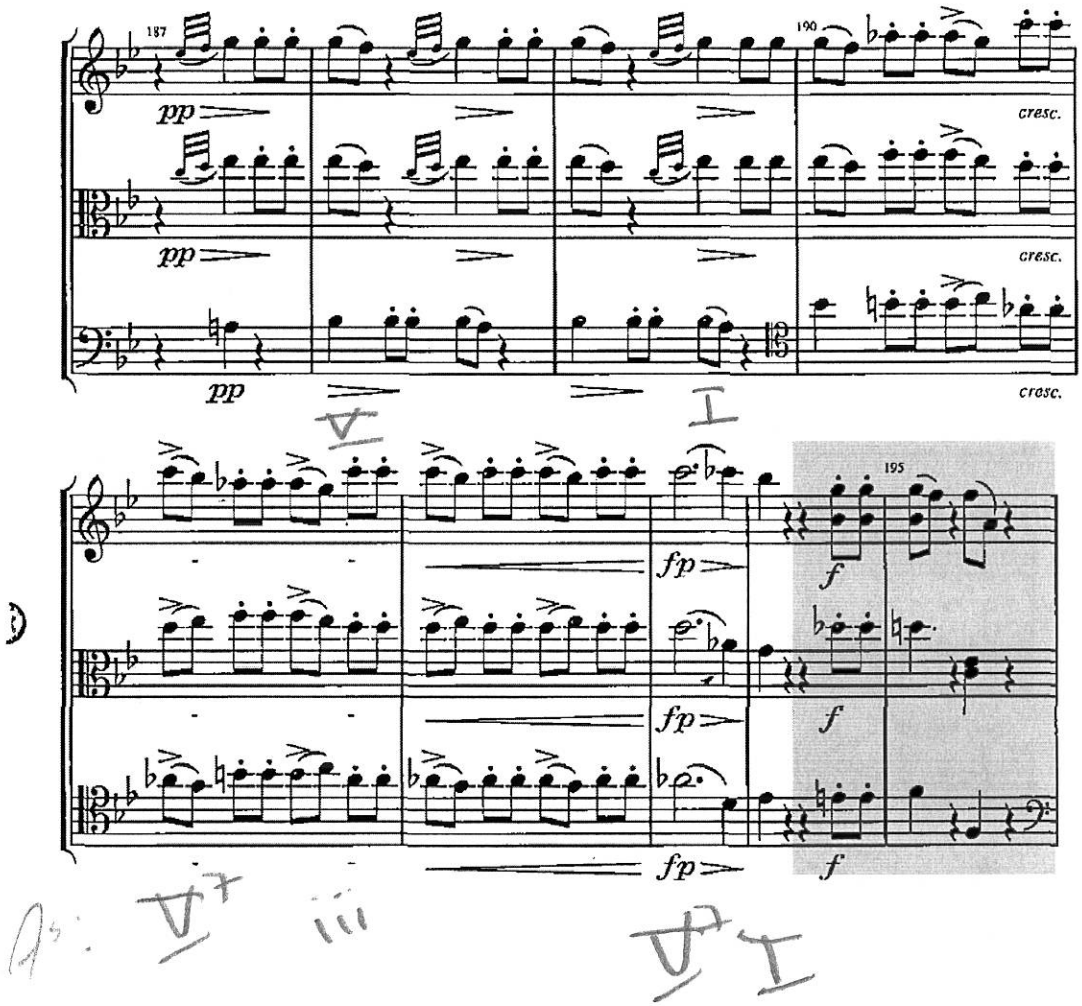
6642

34a: Schubert, "Erlkonig", mm. 134-48



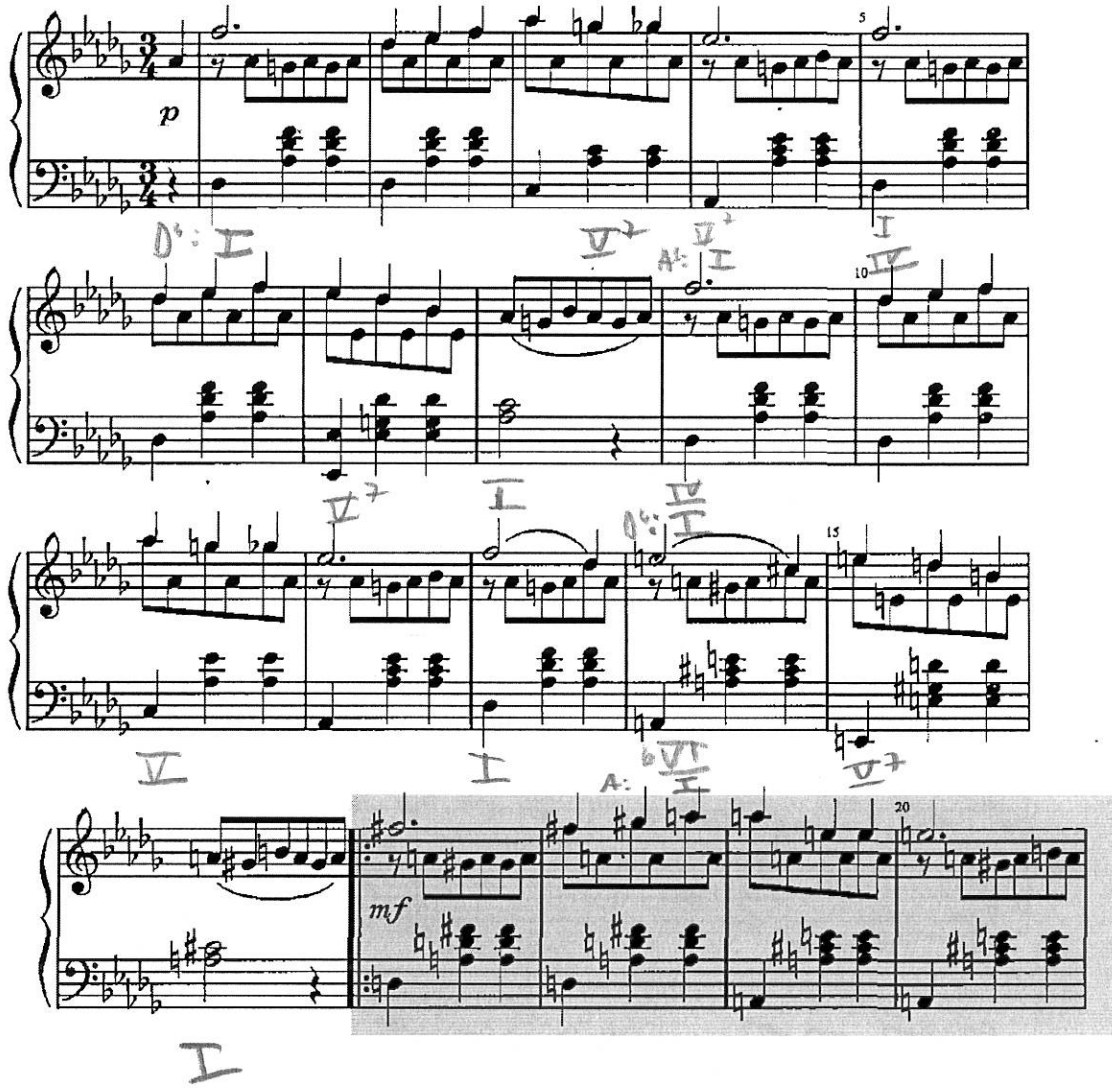
7299

37a: Schubert, String Trio D. 471



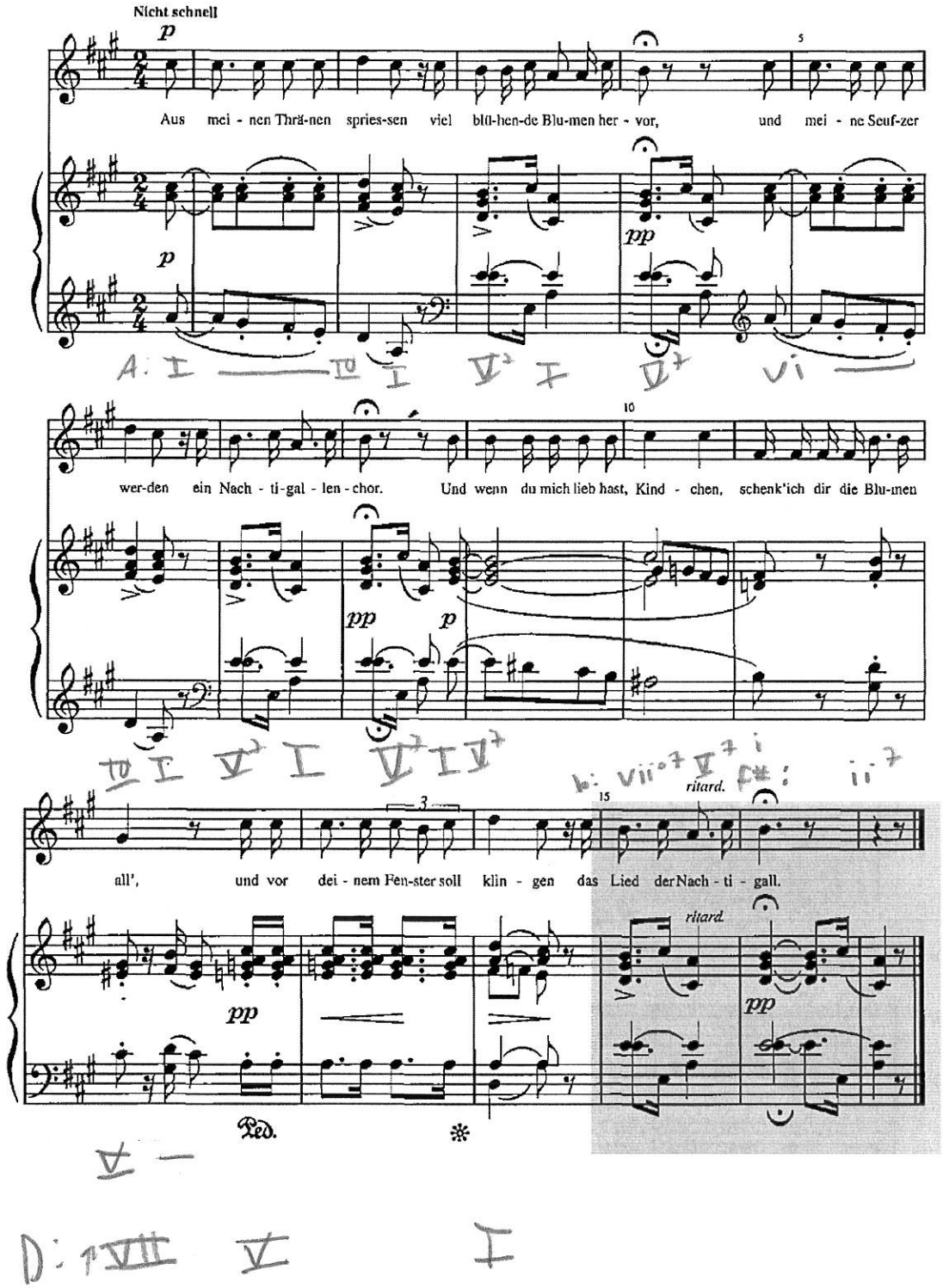
1454

38a: Schubert, Originaltaenze, op. 9, no. 14.



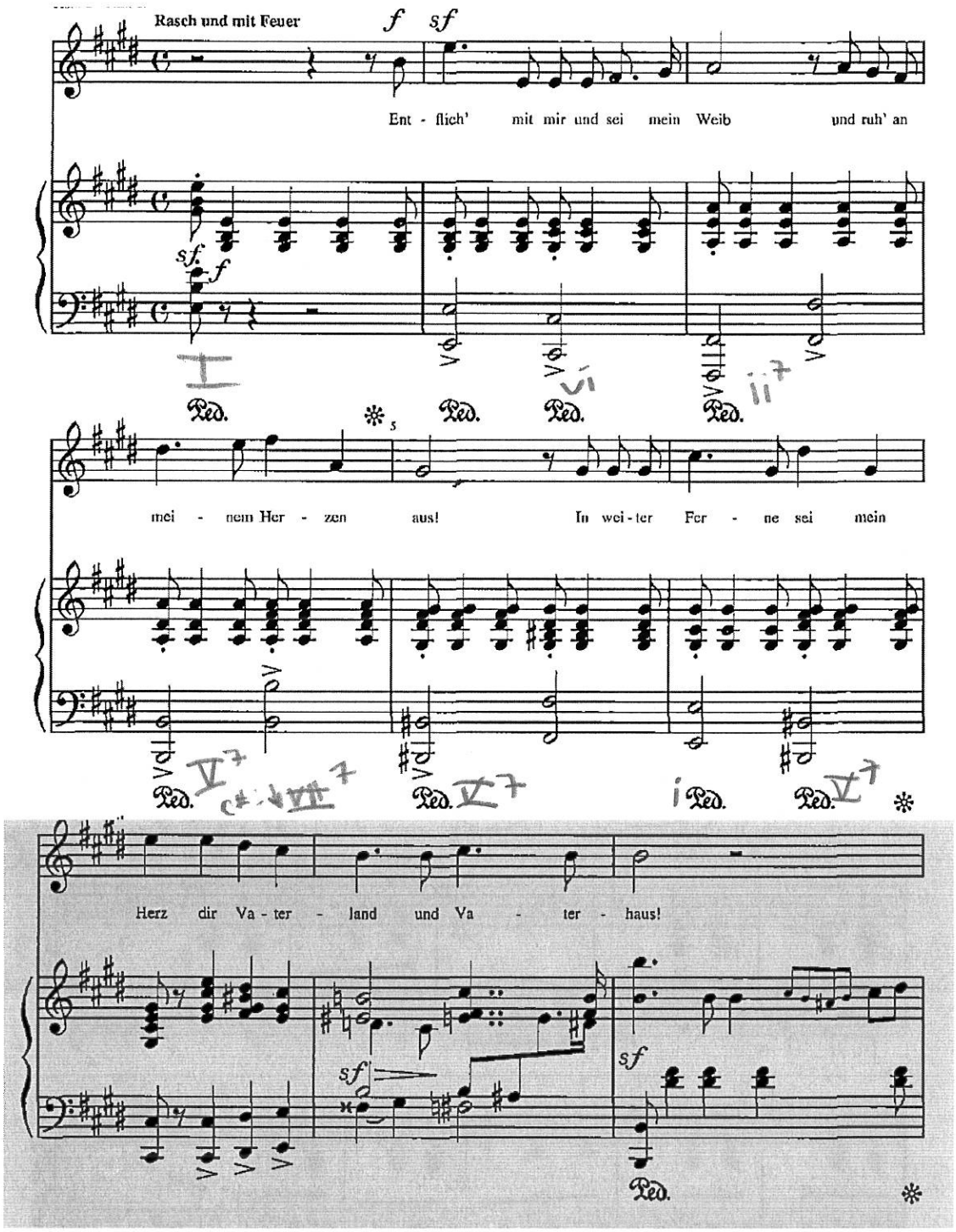
0080

41a: Schumann, "Aus meinen Thraenen spriessen," op. 48, no. 2



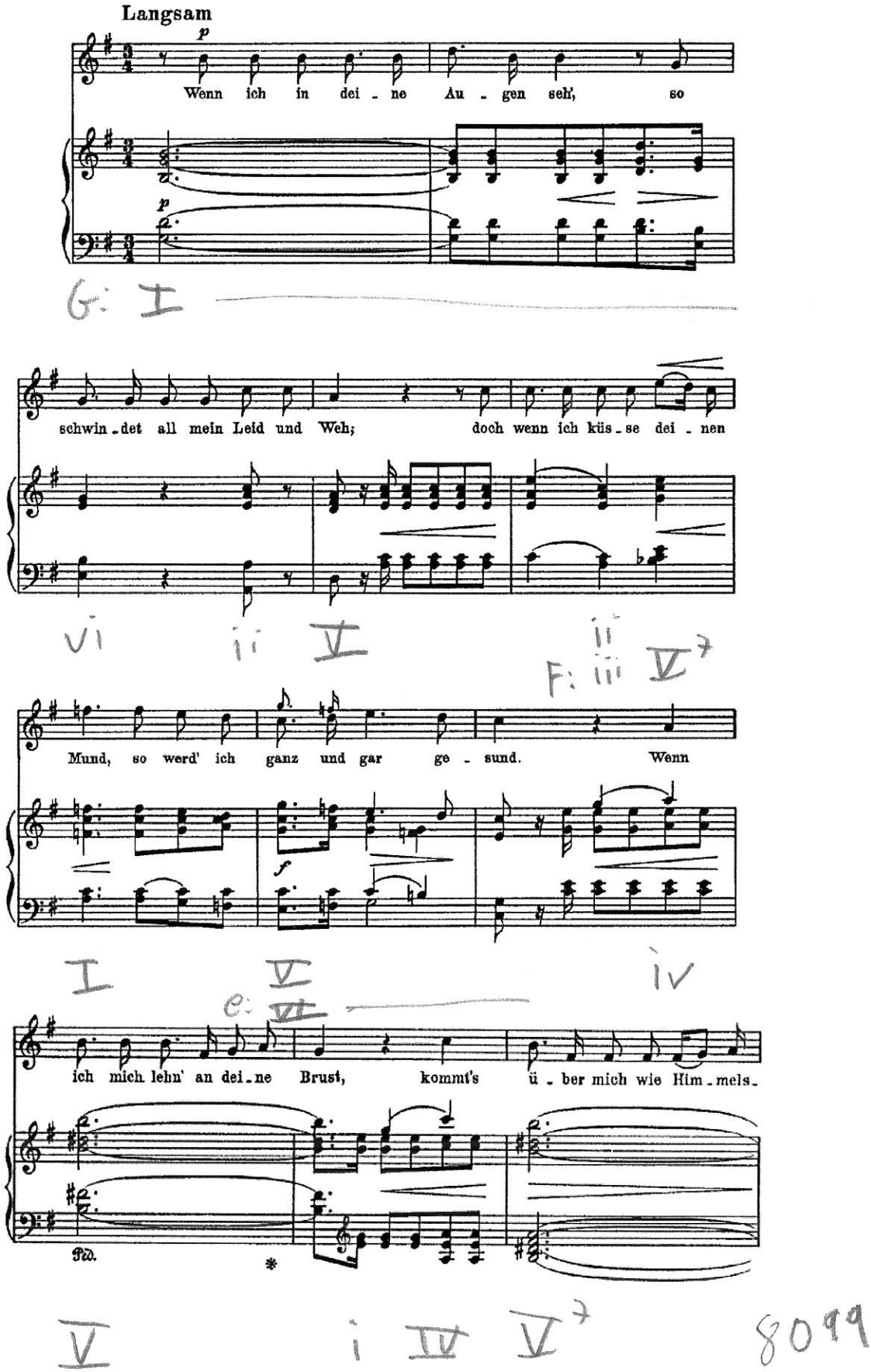
5566

42a: Schumann, Tragoedie, op. 136, no. 3



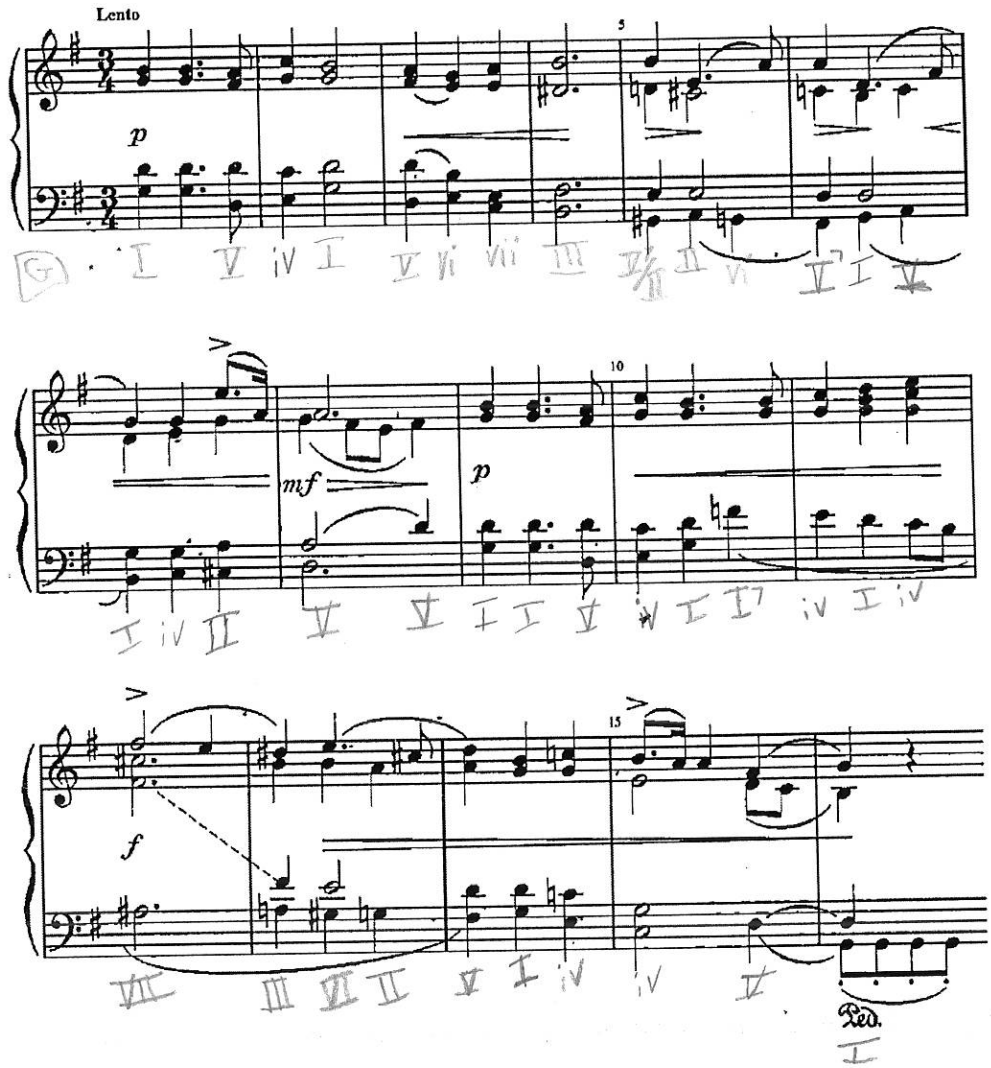
9742

43a: Schumann, "Wenn ich in deine Augen seh'"



8099

44a: Tchaikovsky, "Morning Prayer," op. 39, no. 1



2485